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1\. Let the common ratio between the angles be x.
The angles will be $3x, 5x, 9x$ & $13x$

$\therefore 3x + 5x + 9x + 13x = 360°$ [Sum of all interior angles of a quadrilateral is 360°]

$30x = 360°$

$x = 12°$

$3x = 3 \times 12 = 36°$

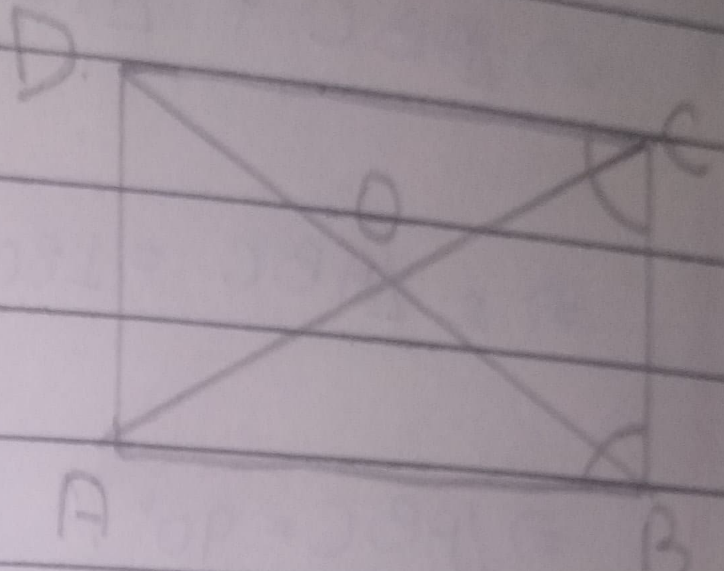
$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

$13x = 13 \times 12 = 156°$

Ex 8.1

2. Given: Let ABCD be a parallelogram where AC = BD



To prove: ABCD is a rectangle

Proof: Rectangle is a parallelogram with one angle 90°. We prove that one of its exterior angles is 90°.

In ΔABC & ΔDCB,

$AB = DC$ [Opposite sides of parallelogram are equal]

$BC = CB$ [Common]

$AC = DB$ (Given)

$\therefore \Delta ABC \cong \Delta DCB$ [SSS congruent]

$\Rightarrow \angle ABC = \angle DCB$

Now,

$AB \parallel DC$ [Oppo. sides of ||gm are parallel]

& BC is a transversal

$\angle ABC + \angle DCB = 180^\circ$

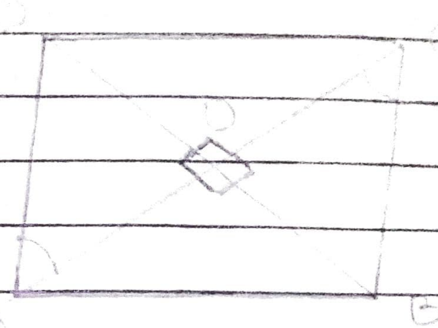
$\Rightarrow \angle ABC + \angle ABC = 180^\circ$

$\Rightarrow 2\angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 90^\circ$

$\therefore$ ABCD is a rectangle

3. ~~Let~~ Given: Let ABCD be a quadrilateral, where diagonals bisect each other



$\therefore$ $OA = OC$, — (1)

and

$OB = OD$, — (2)

And they bisect at right angles

So, $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$ — (3)

To prove: ABCD a rhombus,

Proof: Rhombus is a parallelogram with all sides equal. We will first prove ABCD is a ||gm & then prove all the sides of ABCD are equal.

In ΔAOD & ΔCOB,

$OA = OC$ (From (1))

$\angle AOD = \angle COB$ (From (3))

$OD = OB$ (From (2))

$\therefore$ $\Delta AOD \cong \Delta COB$ (SAS congruence rule)

$\Rightarrow$ $\angle OAD = \angle OCB$ (CPCT)

For sides AD & BC with transversal AC,
$\angle OAD$ & $\angle OCB$ are alternate angles, and they are equal,
So, AD ∥ BC

Similarly, AB ∥ DC

Now, In ABCD, AD ∥ BC & AB ∥ DC
Since opposites sides of ABCD are parallel,
⇒ ABCD is a parallelogram.

In ΔAOB & ΔAOD
OB = OD (given)
$\angle AOB = \angle AOD = 90°$ (given)
AO = AO (common)
$\therefore \Delta AOB \cong \Delta AOD$ (SAS)
⇒ AB = AD [CPCT]

But, AB = DC [ABCD is ∥gm]

AD = BC
AB = BC = CD = DA
ABCD is a rhombus