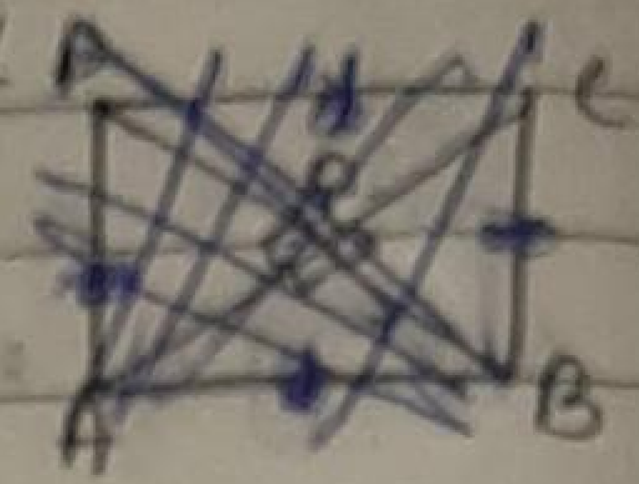
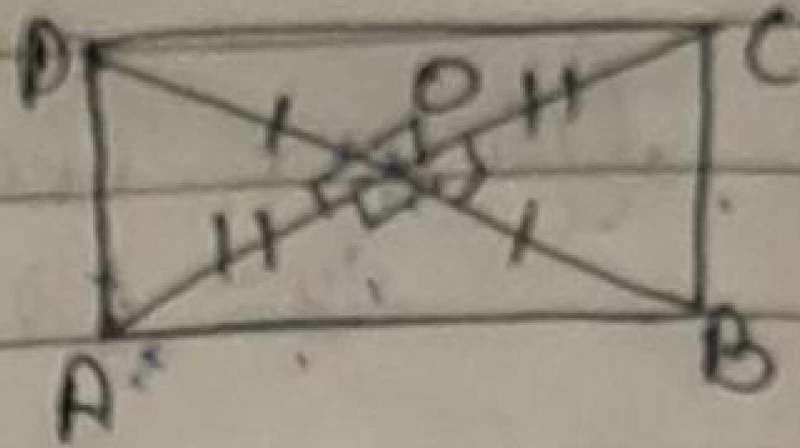
8.1

(5) Given :- $AC = BD$ and bisect each other at $90°$.



To prove :- ABCD is a sq



Proof :- In ΔAOB and AOD,

$AO = AO$ (Common side)

$OB = OD$ (Given)

$\angle AOB = \angle AOD$ ($90°$ each)

$\therefore \Delta AOB \cong \Delta AOD$ (by SAS)

$\therefore AD = AB$ (cpct) $-$ (1)

Similarly,

$\Delta AOB \cong \Delta BOC$, (SAS)

$\therefore AB = BC$ $-$ (cpct) $-$ (2)

In ΔDOC and ΔBOC,

$DO = BO$ (Given)

$OC = OC$ (Common side)

$\angle DOC = \angle BOC$ ($90°$ each)

$\therefore \Delta DOC \cong \Delta BOC$ (by SAS)

$\therefore DC = BC$ (cpct) $-$ (3)

From (1), (2) & (3),

$AD = AB = BC = DC$

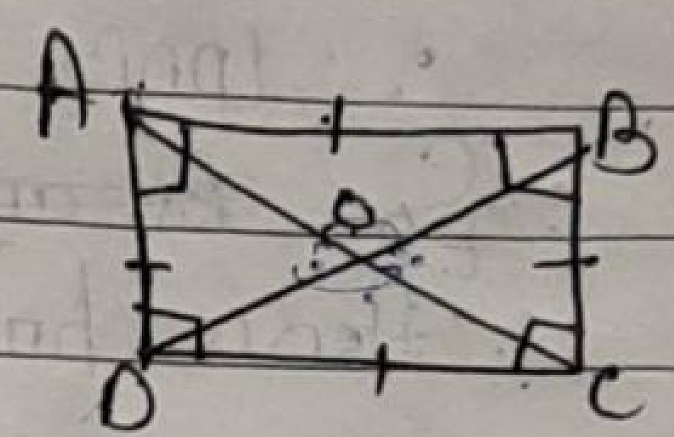
All the sides are equal.

$\therefore$ Hence proved

AB = BC (cpct) — ③

From ①, ② and ③
AD = AB = BC = DC
So, ABCD is a Rhombus.
∴ Hence proved

④ ~~Given diagonals of sq are = bi~~



To prove – AC and BD bisects at 90°

Proof :- In ΔADC and ΔBDC,
∠ADC = ∠BCD (90° each)
AD = BC (Given)
DC = DC (Common Side)
∴ By SAS, ΔADC ≅ ΔBDC.
AC = BD (cpct) — ①
In ΔAOB & ΔDOC,
AB = DC (Given)
∠OAB = ∠OCD (AIA)
∠ABO = ∠ODC (AIA)
∴ By ASA Congruence, ΔAOB ≅ ΔDOC.
OA = OC (cpct) – ②
OD = OB (cpct) – ③
~~∠AOB = ∠COD (cpct) – ④~~

Similarly, taking ΔAOD and BOC,
~~∠AOD =~~
∴ ΔAOD ≅ ΔBOC, by ASA congruence.
∠AOD = ∠BOC (cpct) – ⑤

From ① ~~and~~ ② and ③

$OA = BO = OC = OD$.

In ΔAOD and ΔODC,

$AD = CD$ (given)

$OC = OA$ (prove above)

$OD = OD$ (common side)

$\therefore \Delta AOD \cong \Delta COD$, by SSS congruence.

$\angle AOD = \angle COD$ (CPCT)

~~But~~, $\angle AOD + \angle COD = 180^\circ$

$\Rightarrow \angle AOD + \angle AOD = 180^\circ$

$\Rightarrow 2\angle AOD = 180^\circ$

$\Rightarrow \angle AOD = 90^\circ$

$\angle AOD = \angle COD = \angle BOC = \angle AOB = 90^\circ$

$\therefore$ Hence proved.