

18 Heron's Formula

18.1

① Let each side of the equilateral triangle be a .

$$s = \frac{a+a+a}{2} = \frac{3a}{2}$$

$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{s(s-a)(s-a)(s-a)} = \sqrt{s(s-a)^3} \\ &= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a\right)^3} \\ &= \sqrt{\frac{3a}{2} \times \left(\frac{a}{2}\right)^3} \\ &= \sqrt{\frac{3a^4}{2^4}} = \frac{\sqrt{3}}{4} a^2 \end{aligned}$$

②

$$\begin{aligned} P &= 180 \\ \Rightarrow a+a+a &= 180 \text{ cm} \\ \Rightarrow 3a &= 180 \text{ cm} \\ \Rightarrow a &= \frac{180}{3} \text{ cm} \\ \Rightarrow a &= 60 \text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \frac{\sqrt{3}}{4} a^2 \\ &= \frac{\sqrt{3}}{4} (60)^2 \text{ cm}^2 \\ &= 900 \sqrt{3} \text{ cm}^2 \end{aligned}$$

③

$$P = 540 \text{ cm}$$

$$\Rightarrow 1810 + 1710 + 2510 = 540$$

$$\Rightarrow 5410 = 540$$

$$s = 270 \text{ cm}$$

$$\Rightarrow c = 10$$

$$a = 120 \text{ cm}$$

$$b = 170 \text{ cm}$$

$$c = 250 \text{ cm}$$

$$\begin{aligned} A &= \sqrt{270(270-120)(270-170)(270-250)} \\ &= 9,000 \text{ cm}^2 \end{aligned}$$

(4) Let cube 10 cm

$$P = 42$$

$$\Rightarrow 18 + 10 + 10 = 42$$

$$\Rightarrow 10 = 14 \text{ cm}$$

$$s = \frac{42}{2} = 21 \text{ cm}$$

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-8)(21-10)(21-14)} \\ &= 21\sqrt{11} \text{ cm}^2 \end{aligned}$$

(5) $a+b+c = \text{perimeter}$

$$\Rightarrow 12+12+c = 30 \text{ cm}$$

$$\Rightarrow 24 + c = 30$$

$$\Rightarrow c = 30 - 24$$

$$\Rightarrow \underline{\underline{c = 6}}$$

$$s = \frac{30}{2} = 15$$

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-12)(15-12)(15-6)} \\ &= \sqrt{3 \times 3 \times 3 \times 3 \times 9} \\ &= 3 \times 3 \sqrt{3 \times 3} \\ &= \underline{\underline{9\sqrt{15} \text{ cm}^2}} \end{aligned}$$