Coordinate geometry

1) (2,0), (2,3)

Distance = $\sqrt{(2-2)^2+(0-3)^2}$

$\sqrt{0+9}$ = 3

2) P(1,4), Q(4,0)

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

PQ = $\sqrt{(1-4)^2+(4-0)^2}$

$\sqrt{(-3)^2+(4)^3}$

= 5

3) Area of triangle = $\frac{1}{2}(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2))$

$\frac{1}{2}\cdot(-5\cdot(p-(-2))+1(-2-1)+4\cdot(1-p))$

$\frac{1}{2}\cdot(-5\cdot(p+2)+1(-3)+4(1-p))$

$\frac{1}{2}\cdot(-5p-10-3+4-4p)$

$\frac{1}{2}(-5p-9-4p)=0$

$\frac{1}{2}(-9p-9)=0$

$9p+9=0$

$p=-1$

4) $A(x_1,y_1)=A(a+b,\ b-a)$

$B(x_2,y_2)=B(a-b,\ a+b)$

$|AB|^2=(x_1-x_2)^2+(y_1-y_2)^2$

$((a+b)-(a-b))^2+((b-a)-(a+b))^2$

$(2b)^2-(-2a)^2$

$4b^2-4a^2=4(a^2-b^2)$

$AB=\sqrt{4(a^2-b^2)}$

$2\sqrt{a^2-b^2}$

5) Distance $=\sqrt{(x-3)^2+(-1-2)^2}=5$

$\sqrt{(x-3)^2+9}=5$

$(x-3)^2+9=25$

$(x-3)^2 = 16$

$x - 3 = \pm 4$

$x = 7$ or $x = -1$

$(1,1), (-2,7)$ & $(3,-3)$

$A = (1,1) = (x_1, y_1)$

$B = (-2,7) = (x_2, y_2)$

$C = (3,-3) = (x_3, y_3)$

By distance formula,

$$\sqrt{(x_3-x_2)^2+(y_3-y_2)^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} + \sqrt{(x_3-x_1)^2+(y_3-x_1)^2}$$

$$\sqrt{(3-(-2))^2+(-3-7)^2} = \sqrt{(-3)^2+(6)^2} + \sqrt{(2)^2+(-4)^2}$$

$$\sqrt{25+100} = \sqrt{9+36} + \sqrt{4+16}$$

$$\sqrt{125} = \sqrt{45} + \sqrt{20}$$

$$5\sqrt{5} = 3\sqrt{5} + 2\sqrt{5}$$

$$5\sqrt{5} = 5\sqrt{5}$$

So, they were collinear.

7) $\frac{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)}{2}$

$(x_1, y_1) = (1, 2)$

$(x_2, y_2) = (0, 0)$

$(x_3, y_3) = (a, b)$

$$\frac{1(0-b)+0\cdot(a-1)+a(2-0)}{2} = 0$$

$-b + 2a = 0$

$2a = b$

8) Area of $\triangle ABC = 0$

$$\frac{1}{2}\left(x_1(y_2-y_3)+x_2\cdot(y_3-y_1)+x_3(y_1-y_2)\right)=0$$

$x_1 = 2, \; y_1 = 3$

$x_2 = 4, \; y_2 = 4$

$x_3 = 6, \; y_3 = -3$

$$\frac{1}{2}\cdot\left(2(4-(-3))+4\cdot(-3-3)+6(3-4)\right)=0$$

$2(k+3)+4(-6)+6(3-k)=0\times 2$

$2k+6-24+18-6k=0$

$2k-6k=-6+24-18$

$-4k=0$

$k=0$

9) Point of origin $=(x_1,y_1)=(0,0)$

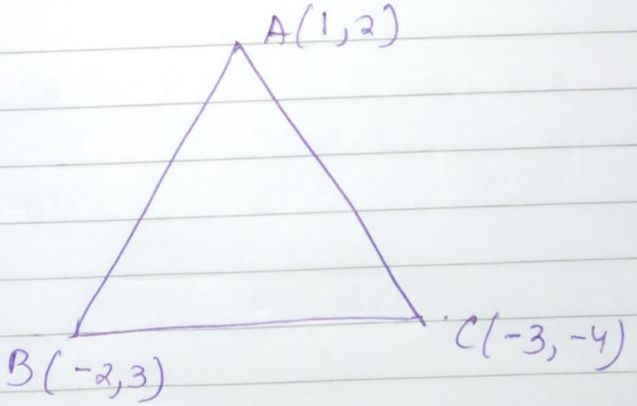
Another point $=(x_2,y_2)=(-3,4)$

$d=\sqrt{(-3-0)^2+(4-0)^2}$

$\sqrt{(-3)^2+(4)^2}$

$\sqrt{9+16}=\sqrt{25}=5$

10)



$AB=\sqrt{(-2-1)^2+(3-2)^2}=\sqrt{9+1}=\sqrt{10}$

$BC=\sqrt{(-2-3)^2+(3+4)^2}=\sqrt{25+49}=\sqrt{74}$

$$AC = \sqrt{(1+3)^2 + (2+4)^2} = \sqrt{16+36} = \sqrt{54}$$