4.07.21 Ch- 8 Quadrilaterals

Ex - 8.1

1 Ans. Let the angles of the Quadrilateral be $3x, 5x, 9x$ and $13x$.

$\therefore \quad 3x + 5x + 9x + 13x = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow 30x = 360°$

$\Rightarrow x = \frac{360°}{30°} = 12°$

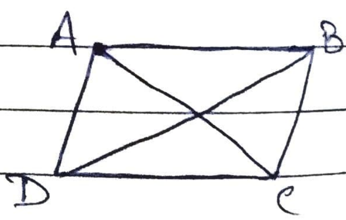
$\therefore \quad 3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

$9x = 9 \times 12° = 108°$

$13x = 13 \times 12° = 156°$

$\therefore$ The required angles of the quadrilateral are $36°$, $60°$, $108°$ and $156°$.



Given,

2 Ans. ~~Let~~ ABCD is a ||gm

AC = BD.

Prove :- ABCD is a rectangle.

Proof :- In ΔABC and ΔDCB,

AC = DB [Given]

AB = DC [Opp. sides of a ||gm]

BC = CB [common].

$\therefore \Delta ABC \cong \Delta DCB$ [SSS].

$\Rightarrow \angle ABC = \angle DCB$ [CPCT] — 1

Now,

$AB \parallel DC$ and BC is a transversal. [∵ ABCD is a ||gm]

∴ $\angle ABC + \angle DCB = 180°$ ---- (2) [co-int. ∠s]

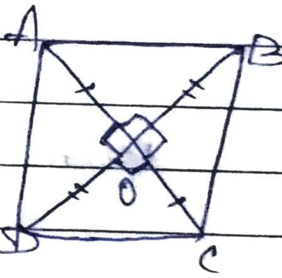
From (1) and (2), we have

$\angle ABC = \angle DCB = 90°$

i.e., ABCD is a parallelogram having an angle equal to 90°.

∴ ABCD is a rectangle.

3.Ans, Given ABCD be a quadrilateral Diagonals AC and BD meet at right angles at O.



Prove :- ABCD is a rhombus.

∴ In $\triangle AOB$ and $\triangle AOD$,

AO = AO [common]

OB = OD [O is the mid-point of BD]

$\angle AOB = \angle AOD$ [right angles]

∴ $\triangle AOB \cong \triangle AOD$ [SAS]

∴ AB = AD [CPCT] ---- (1)

Similarly, AB = BC ---- (2)

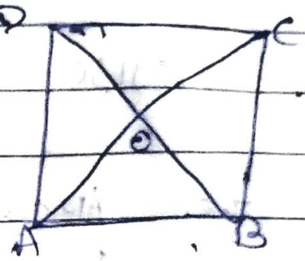
BC = DA ---- (3)

CD = DA ---- (4)

∴ From (1), (2), (3) & (4), we get.

$AB = BC = CD = DA$

Thus, the quadrilateral ABCD is a rhombus.
Alternatively: ABCD can be proved first a ||gm then proving one pair of adjacent sides equal will result in rhombus.



4 Ans, Given,
ABCD is a Square
Its diagonal AC and BD meet at O

Prove,
It's diagonals are equal and bisect each-other at right angle.

Proof:- (i) In $\triangle ABC$ and $\triangle BAD$,
$AB = BA$ [common]
$BC = AD$ [Sides of a Square ABCD]
$\angle ABC = \angle BAD$ [Right angles]
$\therefore \triangle ABC \cong \triangle BAD$ [SAS]
$AC = BD$ [By CPCT] .... (1)

(ii) AD || BC and AC is a ~~transversal~~ transversal [∵ A Square is a ||gm]
$\therefore \angle 1 = \angle 3$ [Alt. int. ∠s]
Similarly,
$\angle 2 = \angle 4$

Now, In $\triangle OAD$ and $\triangle OCB$,

$AD = CB$ [Sides of a Square ABCD].

$\angle 1 = \angle 3$ [Proved]
$\angle 2 = \angle 4$ [Proved]

$\therefore$ $\triangle OAB \cong \triangle OCB$ [ASA]

$\Rightarrow OA = OC$ and $OD = OB$ [CPCT]

i.e. the diagonals AC and BD bisect each other at O. ---- (2)

(iii) In $\triangle OBA$ and $\triangle ODA$,
$OB = OD$ [Proved]
$BA = DA$ [Sides of a Square ABCD]
$OA = OA$ [common]

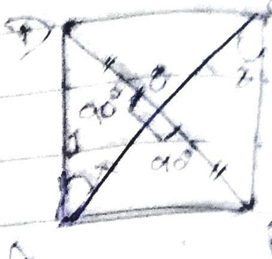
$\therefore$ $\triangle OBA \cong \triangle ODA$ [SSS]
$\Rightarrow \angle AOB = \angle AOD$ [By CPCT] ---- (3)

$\because$ $\angle AOB$ and $\angle AOD$ form a linear pair.
$\therefore$ $\angle AOB + \angle AOD = 180°$
$\therefore$ $\angle AOB = \angle AOD = 90°$ [By (3)]
$\Rightarrow AC \perp BD$ ---- (4)

From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles

5 Ans, Given,

ABCD is a quadrilateral. Diagonals AC & BD are equal and bisect each other at right angles.



Prove :- ABCD is a Square

Proof :- In ΔAOD and ΔAOB,

$\angle AOD = \angle AOB$ [right angles]

$AO = OA$ [Common]

$OD = OB$ [$\because$ O is the midpoint of BD]

$\therefore \Delta AOD \cong \Delta AOB$ [SAS]

$\Rightarrow AD = AB$ [CPCT] ...... (1)

Similarly, we have.

$AB = BC$ ...... (2)

$BC = CD$ ...... (3)

$CD = DA$ ...... (4)

From (1), (2), (3) and (4),

$AB = BC = CD = DA$,

$\therefore$ Quadrilateral ABCD have all sides equal

In ΔAOD and ΔCOB, we have.

$AO = CO$ [Given]

$OD = OB$ [Given]

~~$\angle AOB$ and~~

$\angle AOD = \angle COB$ [Vert. opp. $\angle$s]

So,

$\Delta AOD \cong \Delta COB$ [SAS]

$\therefore \angle 1$ and $\angle 2$ [CPCT]

[But they form a pair of alternate int. $\angle$s].

$\therefore AD \parallel BC$

Similarly, $AB \parallel DC$

$\therefore ABCD$ is a ||gm

$\because$ ||gm having all its sides equal is a rhombus.

$\therefore ABCD$ is a rhombus.

Now, In ΔABC and ΔBAD

$AC = BD$ (given)

$BC = AD$ [Proved]

$AB = BA$ [common]

$\therefore \Delta ABC \cong \Delta BAD$ [SSS]

$\therefore \angle ABC = \angle BAD$ [CPCT] --- (5)

Since, $AD \parallel BC$ and AB is a transversal.

$\therefore \angle ABC + \angle BAD = 180°$ ... (6) [co-int $\angle$s]

$\Rightarrow \angle ABC = \angle BAD = 90°$ [By (5) & (6)]

So, Rhombus $ABCD$ is having one angle equal to $90°$.

Thus,

$ABCD$ is a Square.