Ex - 8.2

2) To prove : PQRS is a rectangle

Construction : Join A & C

Proof :

ΔADC

$SR \parallel AC$

$SR = \frac{1}{2} AC$

ΔBAC

$PQ \parallel AC$

$PQ = \frac{1}{2} AC$

So, $SR = PQ$

$SR \parallel PQ$

So, SRQP is a ∥gm

AQ

If one angle of a parallelogram is 90° then all ~~cong~~ other angles are equal.

So it is a rectangle.

1) (i) In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ — (1)

[Mid-point theorem]

In ΔADC, R is the mid-point of CD and S is the mid-point of BC.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ — (2)

[Mid-point theorem]

In ΔADC, R is the mid-point of CD and S is the mid-point of AD

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ — (2)

(ii) From (1) and (2) we get

$PQ \parallel SR$ and $PQ = SR$

(iii) Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is equal and parallel

$\therefore$ PQRS is a parallelogram

3.) In $\triangle ABC$, P and Q are the mid-points of the sides AB and BC

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ —(i) [Mid point the]

Similarly, in $\triangle ADC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ —(ii)

From (i) and (ii), we get
$PQ \parallel SR$ and $PQ = SR$

Now In quadrilateral PQRS its one pair sides PQ and SR is parallel and equal

$\therefore$ PQRS is a parallelogram.

Now $AD = BC$ —(iv)

[opposite side of a rectangle ABCD]

$\therefore \frac{1}{2} AD = \frac{1}{2} BC$

$AS = BQ$

In $\triangle APS$ and $\triangle BPQ$

$AP = BP$

$AS = BQ$

$\angle PAS = \angle PBQ$

$\triangle APS \cong \triangle BPQ$

$PS = PQ$ Proved

To prove –

4.) F is the mid-point of BC.

Proof : AB || DC and EF || AB

$\Rightarrow$ AB, EF and DC are ||

Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

$\therefore$ Intercepts made by those parallel lines on transversal BC are also i.e, BF = FC

$\Rightarrow$ F is the mid-point BC.