4. In $\triangle ABC$, $DE \parallel AC$

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ---- (i)

In $\triangle ABE$, $DF \parallel AE$

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ---- (ii)

From eq (i) and (ii), we have

$\therefore \frac{BE}{EC} = \frac{BF}{FE}$

5. In $\triangle PQS$, $DE \parallel QS$

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \text{---(i)}$$

In $\triangle PQR$, $DF \parallel QR$

$$\frac{PF}{FR} = \frac{PD}{DO}$$

From eq (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$

6. In $\triangle PQQ$, $AB \parallel PQ$

$$\frac{QP}{PA} = \frac{QQ}{QB} \quad \text{---(i)}$$

In $\triangle PQR$, $AC \parallel PR$

$$\frac{QP}{PA} = \frac{QR}{RC} \quad \text{---(ii)}$$

$$\therefore \frac{QQ}{QB} = \frac{QR}{RC} \qquad \therefore BC \parallel QR$$

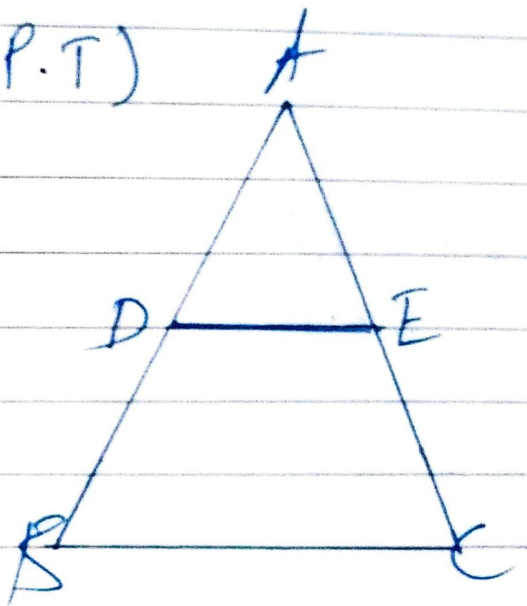
7. Given :- $DE \parallel BC$

To prove :- $AE = EC$

Proof :- In $\triangle ABC$,

$DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By B.P.T)




$\because$ But $AD = DB$

$\therefore \frac{AD}{DB} = 1$

$1 = \frac{AE}{EC}$

$\therefore AE = EC$

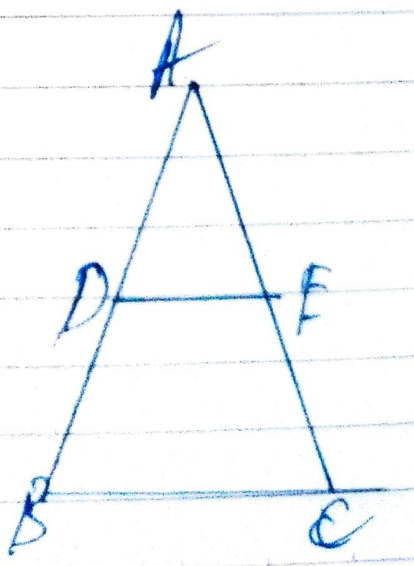
8. In $\triangle ABC$,

$AD = DB$ and $AE = EC$

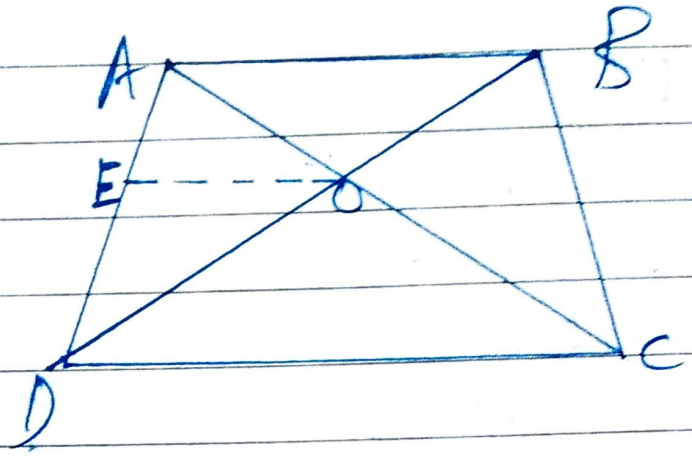
$\frac{AD}{DB} = 1$ and $\frac{AE}{AC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{AC}$

$\therefore DE \parallel BC$



9.




In $\triangle ABD$, $EO \parallel DC$

$DC \parallel AB$

$EO \parallel AB$

$\therefore \frac{AE}{ED} = \frac{BO}{DO}$ ---- (i)

In $\triangle ADC$, $EO \parallel DC$

$\frac{AE}{ED} = \frac{AO}{CO}$ ---- (ii)

$\therefore$ From eq (i) and (ii), we get

$\frac{BO}{DO} = \frac{AO}{CO}$ or $\frac{AO}{BO} = \frac{CO}{DO}$

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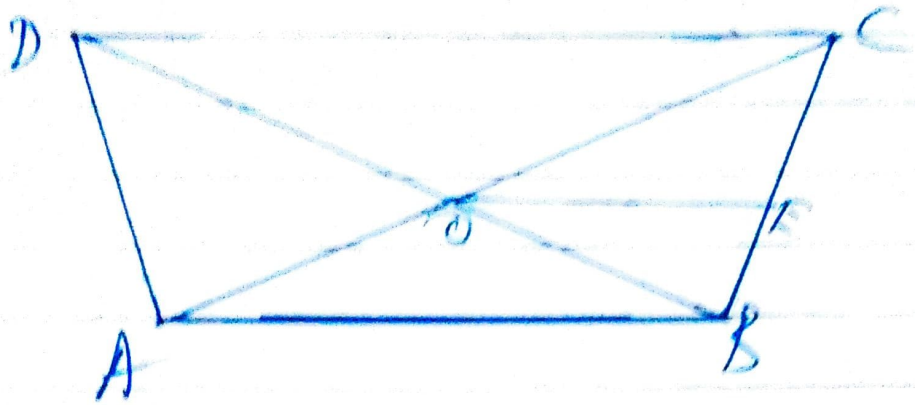


In $\triangle ABC$, $OE \parallel AB$

$$\frac{AO}{OC} = \frac{BE}{EC} \quad \ldots (i)$$

And, $$\frac{AO}{OC} = \frac{BO}{DO} \quad \ldots (ii)$$

$\therefore$ From eq (i) and (ii) we get

$$\frac{BE}{EC} = \frac{BO}{DO}$$

$\therefore$ $OE \parallel DC$

$OE \parallel AB$ and $OE \parallel DC$

$\therefore$ $AB \parallel DC$.

$\therefore$ ABCD is a trapezium.