5. Given :- D, E, F are midpoints of sides BC, CA, AB.
Proof :- D and E are midpoint of side BC and CA.

$\because DE = \frac{1}{2}AB, \quad EF = \frac{1}{2}BC, \quad DF = \frac{1}{2}AC$

$\because \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{1}{2}$

$\because \Delta DEF \sim \Delta ABC$

$\because \frac{ar\ \Delta DEF}{ar\ \Delta ABC} = \frac{DE^2}{AB^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\because ar\ \Delta DEF : ar\ \Delta ABC = 1:4$

6. Given :- $\Delta ABC \sim \Delta DEF$, AP and DQ are median.
To prove :- $\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AP^2}{DQ^2}$

Proof :- $\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AB^2}{DE^2}$

$\because \Delta ABC \sim \Delta DEF$

$\because \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$

$\because \frac{AB}{DE} = \frac{BP}{EQ}$ --- (i)

$\angle B = \angle E$

$\therefore \Delta ABP \sim \Delta DEQ$ (SAS)

$\frac{BP}{EQ} = \frac{AP}{DQ}$ — (ii)

$\therefore$ From eq (i) and (ii), $\frac{AB}{DE} = \frac{AP}{DQ}$

$\therefore \frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AP^2}{DQ^2}$

7. $AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

$AC = \sqrt{2}a$

$\because \Delta PAD \sim \Delta QAC$ (AA)

$\frac{ar\ \Delta PAD}{ar\ \Delta QAC} = \frac{AD^2}{AC^2} = \frac{a^2}{(\sqrt{2}a)^2} = \frac{1}{2}$

$\therefore ar\ \Delta PAD = \frac{1}{2}\ ar\ \Delta QAC$

8. Let $AB = BC = CA = a$

$BD = \frac{1}{2}a$

$\because \Delta ABC \sim \Delta BDE$

$\therefore \frac{ar\ \Delta ABC}{ar\ \Delta BDE} = \frac{AB^2}{BD^2} = \frac{a^2}{(\frac{1}{2}a)^2} = \frac{a^2}{\frac{1}{4}a^2} = \frac{4}{1}$ or $4:1$

9. Ratio of areas of triangle $= (\frac{4}{9})^2 = \frac{16}{81}$ or $16:81$