Quadratic Equation.

Exercise 4.3

Q1)(i) Given : $x^2 - 7x + 3 = 0$

$\Rightarrow 2\left[x^2 - \frac{7}{2}x + \frac{3}{2}\right] = 0$

$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$\Rightarrow (x)^2 - \left(\frac{7}{2}x\right) + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49}{16} - \frac{3}{2}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49 - 24}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{25}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2 = 0$

$\Rightarrow \left(x - \frac{7}{4} + \frac{5}{4}\right)\left(x - \frac{7}{4} - \frac{5}{4}\right) = 0$

$\Rightarrow \left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right) = 0.$

$\Rightarrow (x - 3)\left(x - \frac{1}{2}\right) = 0$

$\Rightarrow x = 3 \quad \text{or} \quad x = \frac{1}{2}.$

(ii) $2x^2+x-4=0$

$\Rightarrow x^2+\frac{x}{2}-2=0$

$\Rightarrow x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{1}{16}-2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-\left(\frac{33}{16}\right)=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-\left(\frac{\sqrt{33}}{4}\right)^2=0.$

$\Rightarrow \left(x+\frac{1}{4}+\frac{\sqrt{33}}{4}\right)\left(x+\frac{1}{4}-\frac{\sqrt{33}}{4}\right)=0$

$\Rightarrow \left(\frac{x+1+\sqrt{33}}{4}\right)\left(x+\frac{1-\sqrt{33}}{4}\right)=0.$

$\Rightarrow x=\frac{-1-\sqrt{33}}{4}$ or $x=\frac{-1+\sqrt{33}}{4}.$

Q2) i) $2x^2-7x+3=0$

$D=b^2-4ac$

$=(-7)^2-4\times2\times3=49-24=25.$

$D>0.$

$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-7)+\sqrt{25}}{2\times2}=\frac{7+5}{4}=\frac{12}{4}=3.$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-7)-\sqrt{25}}{2\times 2} = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}.$$

(ii) $2x^2 - x + 4 = 0.$

$D = b^2 - 4ac$

$= (1)^2 - 4 \times 2(-4) = 1 + 32 = 33.$

$D > 0.$

$$\alpha = \frac{+b+\sqrt{D}}{2a} = \frac{-1+\sqrt{33}}{4}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-1-\sqrt{33}}{4}.$$

Q3) i) Given : $x - \frac{1}{x} = 3$

Multiplying both sides by x, we get :

$$x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0.$$

This is a quadratic equation.

Here, $a = 1$, $b = -3$ and $c = -1$.

$$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}.$$

$$= \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}.$$

Either $x = \frac{3+\sqrt{13}}{2}$ or $x = \frac{3-\sqrt{13}}{2}$.

4) Let the present age of Rehman be x years.
3 years ago Rehman's age was $= (x-3)$ years.
5 years from now Rehman's age will be $= (x+5)$ yrs.

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$$

$$\Rightarrow 6x+6 = x^2+2x-15 \Rightarrow x^2-4x-21 = 0.$$

$\Rightarrow (x+3)(x-7) = 0$

$\Rightarrow x+3 = 0$ or $x-7 = 0$.

$\Rightarrow x = 7$ or -3 (-3 is rejected)

Hence, present age of Rehman is 7 years.

5) Let Shefali's marks in Maths be x.
Then, her marks in English will be $30-x$.
According to the question, we have:

$$(x+2)(30-x-3) = 210$$

$\Rightarrow (x+2)(27-x) = 210 \Rightarrow -x^2+25x+54 = 210 \Rightarrow x^2-25x+156 = 0$

$\Rightarrow x^2-12x-13x+156 = 0 \Rightarrow (x-12)(x-13) = 0$

$\Rightarrow x = 12$ or $x = 13 = 0$.

6) $x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$.

$\Rightarrow x^2 - 60x - 2700 = 0$.

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$.

$\Rightarrow x^2(x-90) + 30(x-90) = 0 \Rightarrow (x+30)(x-90) = 0$

$\Rightarrow x + 30 = 0$ or $x - 90 = 0 \Rightarrow x = -30$ or $x = 90$.

Shorter side $= 90$ m.

Longer side $= 90 + 30 = 120$ m.