4. Check whether $(5,-2)$, $(6,4)$ and $(7,-2)$ are the vertices of an isosceles triangle.

– Let points be $A(5,-2)$, $B(6,4)$ and $C(7,-2)$.

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4+0} = 2$$

Here, $AB = BC$

$\therefore \triangle ABC$ is an isosceles triangle.

5. Let the points along with coordinates be $A(3,4)$, $B(6,7)$, $C(9,4)$ and $D(6,1)$

Then by distance formula,

$$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$$

$$DA = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{18} = 3\sqrt{2}$$

Also, diagonal $AC = \sqrt{(9-3)^2 + (4-4)^2}$

$$= \sqrt{(6)^2 + (0)^2} = \sqrt{36} = 6$$

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and diagonal $BD = \sqrt{(6-6)^2 + (1+7)^2}$

$= \sqrt{(0)^2 + (-6)^2} = \sqrt{36} = 6$

$= AB = BC = CD = DA \; 3\sqrt{2}$
and diagonals $AC = BD = 6$

Thus, ABCD is a square and Champa is correct.

6.
(i) Let points be $A(-1,-2)$; $B(1,0)$, $C(-1,2)$ and $D(-3,0)$.

The distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
$AD = \sqrt{(-3+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = \sqrt{16} = 4$
$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

Here, $AC = BD$, $AB = BC = CD = AD$
Hence, the quadrilateral ABCD is a square.

(iii) Let points be $A(4,5)$, $B(7,6)$, $C(4,3)$ and $D(1,2)$.

$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$
$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = 3\sqrt{2}$
$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$
$AD = \sqrt{(1-4)^2 + (2-5)^2} = \sqrt{9+9} = 3\sqrt{2}$

$AC = \sqrt{(9-4)^2 + (3-5)^2} = \sqrt{4} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

Here, $AB = CD$, $BC = AD$

and $AC \neq BD$

The quadrilateral ABCD is a parallelogram

7. Let $A(2,-5)$ and $B(-2,9)$ be the given points

Also let $P(x,0)$ be the point on x axis such that

$PA = PB$

Then $PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$\Rightarrow (x-2)^2 - (x+2)^2 = 81 - 25$

$\Rightarrow (x-2+x+2)(x-2-x-2) = 56$

$\Rightarrow (2x)(-4) = 56$

$\Rightarrow -8x = 56$

$\Rightarrow x = -7$

Hence, the required point is $(-7,0)$

8. Points $P(2,-3)$, $Q(10,y)$ and $PQ = 10$ unit

The distance between two points is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = PQ \Rightarrow \sqrt{(10-2)^2+(y+3)^2} = 10$

$= 64 + y^2 + 9 + 6y = 100 \Rightarrow y^2 + 6y + 73 - 100 = 0$

$= y^2 + 6y - 27 = 0 \Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0 \Rightarrow (y-3)(y+9) = 0$

$= y - 3 = 0$ or $y + 9 = 0$

$\Rightarrow y = 3$ or -9

9. Given that $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$

$\therefore QP = QR \Rightarrow QP^2 = QR^2$

$= (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$= 25 + 16 = x^2 + 25$

$= x^2 = 16 \Rightarrow x = \pm 4$

$QR = \sqrt{(x-0)^2+(6-1)^2} = \sqrt{x^2+5^2}$

$= \sqrt{(4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(x-5)^2 + (6+3)^2}$

$= \sqrt{(4-5)^2 + (6+3)^2}$

$= \sqrt{(-1)^2 + (9)^2} = \sqrt{1+81} = \sqrt{82}$

Also, $PR = \sqrt{(-4-5)^2 + (6+3)^2}$.

$= \sqrt{(-9)^2 + (9)^2} = \sqrt{162} = 9\sqrt{2}$

Hence, $QR = \sqrt{41}$ and $PR = \sqrt{82}, 9\sqrt{2}$.

10. Points A(3,6) and B(-3,4) are equidistant from point P(x, y)

$AP = BP = \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y$

$= x^2 + 9 + 6x + y^2 + 16 - 8y$

$\Rightarrow -6x - 6x - 12y + 8y + 45 - 25 = 0$

$\Rightarrow 12x + 4y + 20 = 0$

Dividing by -4, we get $3x + y - 5 = 0$