

Current Electricity

① $E = 12V$, $r = 0.4\Omega$

the current drawn from the battery will be maximum when the external resistance in the circuit is zero. i.e, $R = 0$

$$I_{\max} = \frac{E}{r} = \frac{12}{0.4} = 30A$$

② $E = 10V$

$r = 3\Omega$

$I = 0.5A$

$$I = \frac{E}{R+r}$$

$$R+r = \frac{E}{I}$$

$$R = \frac{E}{I} - r = \frac{10}{0.5} - 3 = 17\Omega$$

Terminal voltage,

$$V = IR = 0.5 \times 17 = 8.5V$$

③

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

} in series.

$E = 12V$

(i) $R_s = R_1 + R_2 + R_3 = 6\Omega$

(ii) Current in circuit, $I = \frac{E}{R} = \frac{12}{6} = 2A$

Potential drop across different resistors:

$$V_1 = IR_1 = 2 \times 1 = 2V$$

$$V_2 = IR_2 = 2 \times 2 = 4V$$

$$V_3 = IR_3 = 2 \times 3 = 6V$$

④ $R_1 = 2\Omega$
 $R_2 = 4\Omega$
 $R_3 = 5\Omega$ } parallel.

$$E = 20V$$

$$\text{v)} \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{19}{20}$$

$$R_p = \frac{20}{19} \Omega$$

vi) Current through resistors:

$$I_1 = \frac{E}{R_1} = \frac{20}{2} = 10A \quad , \quad I_2 = \frac{E}{R_2} = \frac{20}{4} = 5A$$

$$I_3 = \frac{E}{R_3} = \frac{20}{5} = 4A$$

⑤ $R_1 = 100\Omega$, $R_2 = 117\Omega$, $t_1 = 27^\circ C$

$$\alpha = 1.70 \times 10^{-4} \text{ } ^\circ C^{-1}$$

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

$$\therefore t_2 - t_1 = \frac{R_2 - R_1}{R_1 \alpha} = \frac{117 - 100}{100 \times 1.70 \times 10^{-4}} = 1000$$

$$t_2 = 1000 + t_1 = 1000 + 27 = 1027^\circ C$$

⑥ $l = 15 \text{ m}$, $A = 6.0 \times 10^{-7} \text{ m}^2$, $R = 5.0 \Omega$

resistivity, $\rho = \frac{RA}{l} = \frac{5.0 \times 6.0 \times 10^{-7}}{15}$

$= 2.0 \times 10^{-7} \Omega \text{ m}$

⑦ $R_1 = 2.1 \Omega$, $t_1 = 27.5^\circ \text{C}$, $R_2 = 2.7 \Omega$, $t_2 = 100^\circ \text{C}$

temp coefficient of resistivity of silver

$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$

$= \frac{2.7 - 2.1}{2.1 (100 - 27.5)}$

$= \frac{0.6}{2.1 \times 72.5}$

$= 0.00394^\circ \text{C}^{-1}$

⑧ $V = 230 \text{ V}$, $I_1 = 3.2 \text{ A}$

$I_2 = 2.8 \text{ A}$, $\alpha = 1.70 \times 10^{-4} \text{ }^\circ \text{C}^{-1}$

Resistance at room temp.

$R_1 = \frac{V}{I_1} = \frac{230}{3.2} = 82.143 \Omega$

$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$

$\therefore t_2 - t_1 = \frac{R_2 - R_1}{R_1 \alpha}$

$= \frac{82.143 - 71.875}{71.875 \times 1.70 \times 10^{-4}}$

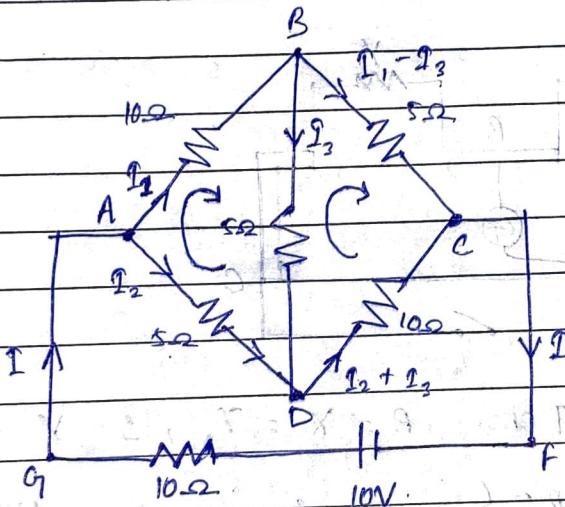
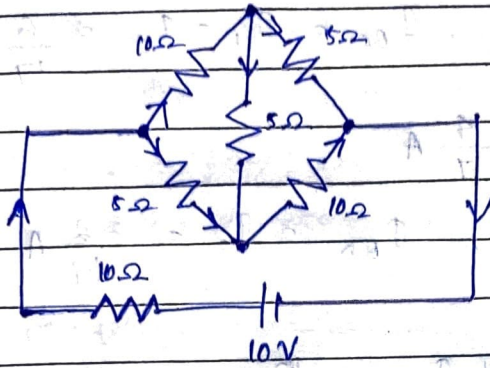
$= \frac{10.268 \times 10^4}{71.875 \times 1.7}$

$= 840.35^\circ \text{C}$

∴ Steady temperature

$$t_2 = 840.35 + 27 = 867.35 \text{ } ^\circ\text{C}.$$

(9)



For loop ABDA,

$$10I_1 + 5I_3 - 5I_2 = 0.$$

For loop BCDB,

$$5(I_1 - I_3) - 10(I_2 + I_3) - 5I_3 = 0.$$

For loop ADCF GA,

$$5I_2 + 10(I_2 + I_3) + 10(I_1 + I_2) = 10 \quad (1)$$

$$10I_1 - 5I_2 + 5I_3 = 0 \quad (2)$$

$$5I_1 - 10I_2 - 20I_3 = 0 \quad (3)$$

$$10I_1 + 25I_2 + 10I_3 = 10 \quad (4)$$

Solving ①, ②, ③.

$$I_1 = \frac{4}{17} \text{ A}, \quad I_2 = \frac{6}{17} \text{ A}, \quad I_3 = -\frac{2}{17} \text{ A}$$

$$I_{AB} = I_1 = \frac{4}{17} \text{ A}, \quad I_{BC} = I_1 - I_3 = \frac{6}{17} \text{ A}$$

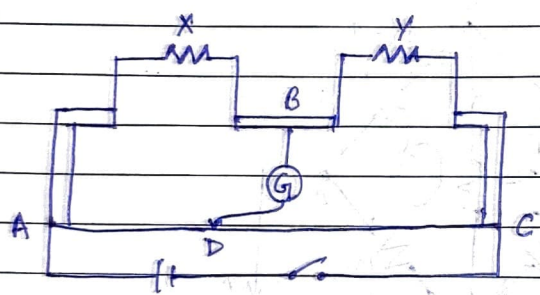
$$I_{DC} = I_2 + I_3 = \frac{4}{17} \text{ A}$$

$$I_{AD} = I_2 = \frac{6}{17} \text{ A}, \quad I_{BD} = I_3 = -\frac{2}{17} \text{ A}$$

Total current

$$I = I_1 + I_2 = \frac{10}{17} \text{ A}$$

⑩



Here $l = 35.9 \text{ cm}$, $R = X = Y$, $S = Y = 12.5 \Omega$.

$$A \Rightarrow S = \frac{100 - l}{l} \times R \quad \therefore 12.5 = \frac{100 - 35.9}{35.9} \times R$$

$$\therefore R = \frac{12.5 \times 35.9}{60.5} = 8.16 \Omega$$

Connections are made by thick copper strips to minimize the resistances of connections which are not accounted for in the above formula -

(ii) When X and Y are interchanged

$$R = Y = 12.5 \Omega, \quad S = X = 8.16 \Omega, \quad l = ?$$

$$S = \frac{100 - l}{l} \times R \quad \therefore 8.16 = \frac{100 - l}{l} \times 12.5$$

$$\text{or } 8.16 L = 1250 - 12.5 L$$

$$L = \frac{1250}{20.66} = 60.5 \Omega \text{ from the end A.}$$

(iii) When the galvanometer and cell are interchanged at the balanced point, the conditions of the balanced bridge are still satisfied and so again the galvanometer will not show any current.

$$\textcircled{11} \quad E = 8.0 \text{ V}$$

$$r = 0.5 \Omega$$

$E = 120 \text{ V}$ dc supply using a series resistor of 15.5Ω .

$$E = 120 - 8.0 = 112 \text{ V}$$

Current in circuit during charging,

$$I = \frac{E}{R+r} = \frac{112}{15.5 + 0.5} = 7 \text{ A}$$

terminal voltage of battery during charging,

$$V = E + Ir = 8.0 + 7 \times 0.5 = 11.5 \text{ V}$$

$$\textcircled{12} \quad E_1 = 1.25 \text{ V}$$

$$l_1 = 35.0 \text{ cm}$$

$$l_2 = 63.0 \text{ cm}$$

$$E_2 = ?$$

$$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

$$\therefore E_2 = \frac{l_2}{l_1} \times E_1 = \frac{63 \times 1.25}{35} = 2.25 \text{ V}$$

(13) $n = 8.5 \times 10^{28} \text{ m}^{-3}$, $l = 3 \text{ m}$
 $A = 2.0 \times 10^{-6} \text{ m}^2$, $e = 1.6 \times 10^{-19} \text{ C}$, $I = 3.0 \text{ A}$

Drift speed,

$$V_d = \frac{I}{enA}$$

$$= \frac{3}{1.6 \times 10^{-19} \times 8.5 \times 10^{28} \times 2 \times 10^{-6}} \text{ m s}^{-1}$$

$$= \frac{3}{16 \times 85 \times 2 \times 10} \text{ m s}^{-1} \approx 1.1 \times 10^{-4} \text{ m s}^{-1}$$

Required time,

$$t = \frac{l}{V_d} = \frac{3}{1.1 \times 10^{-4} \text{ s}} = 2.73 \times 10^4 \text{ s}$$

$$\approx 7.57 \text{ h}$$

(14) Surface charge density

$$\sigma = 10^{-9} \text{ C m}^{-2}$$

Radius of the earth

$$R = 6.37 \times 10^6 \text{ m}$$

Current, $I = 1800 \text{ A}$

Total charge of the globe,

$$Q = \text{surface area} \times \sigma = 4\pi R^2 \sigma$$

$$= 4 \times 3.14 \times (6.37 \times 10^6)^2 \times 10^{-9}$$

$$= 509.65 \times 10^3 \text{ C}$$

Required time,

$$t = \frac{Q}{I} = \frac{509.65 \times 10^3}{1800} = 283.13 \text{ s}$$

$$\approx 283 \text{ s}$$

(15) $E = 2V$, $r = 0.015 \Omega$, $R = 8.5 \Omega$, $n = 6$

When the cells are joined in series, the current is

$$I = \frac{nE}{R + nr} = \frac{6 \times 2}{8.5 + 6 \times 0.015} = \frac{12}{8.59} \text{ A} \approx 1.4 \text{ A}$$

Terminal Voltage,

$$V = IR = 1.4 \times 8.5 = 11.9 \text{ V}$$

(6) Here $E = 1.9 \text{ V}$, $r = 380 \Omega$.

$$I_{\text{max}} = \frac{E}{r} = \frac{1.9}{380} \text{ A} = 0.005 \text{ A}$$

The secondary cell cannot drive the starting motor of a car because that requires a large current of about 100 A for a few seconds.

(16) $\rho_{\text{Al}} = 2.63 \times 10^{-8} \Omega \text{ m}$, $\rho_{\text{Cu}} = 1.72 \times 10^{-8} \Omega \text{ m}$

relative density of Al = 2.7 and that of Cu = 8.9.

$$\text{Mass} = \text{volume} \times \text{density} = A \cdot l \cdot d$$

$$= \frac{\rho l}{R} \cdot l \cdot d = \frac{\rho d l^2}{R} \quad \left[\because R = \rho \frac{l}{A} \right]$$

As the two wires are of equal length and have the same resistance, their mass ratio will be

$$\frac{m_{\text{Cu}}}{m_{\text{Al}}} = \frac{\rho_{\text{Cu}} d_{\text{Cu}}}{\rho_{\text{Al}} d_{\text{Al}}} = \frac{1.72 \times 10^{-8} \times 8.9}{2.63 \times 10^{-8} \times 2.7} = 2.1558 \approx 2.2$$

copper wire is 2.2 times heavier than aluminium wire.

Since aluminium is lighter, it is preferred for long suspension of cables otherwise heavy cable may sag down due to its own weight.

(18) (a) Only current is constant because, it is given to be steady. Other quantities: current density, electric field and drift speed vary inversely with area of cross-section.

(b) No, Ohm's law is not universally applicable for all conducting elements. Examples of non-ohmic elements, are vacuum diode, semiconductor diode, thyristor, gas discharge tube, electrolytic solutions, etc.

(c) The maximum current that can be drawn from a voltage supply is given by

$$I_{\max} = \frac{E}{\rho}$$

I_{\max} will be large if ρ is small.

(d) If the internal resistance is not very large, then the current will exceed the safety limits in case the circuit is short-circuited accidentally.

(19) (a) Alloys of metals usually have greater resistivity than that of their constituent metals.

(b) Alloys usually have much lower temperature coefficient of resistance than pure metals.

(c) The resistivity of the alloy manganin is nearly independent of temperature.

(d) The resistivity of a typical insulator is greater than that of a metal by a factor of the order of 10^{22} .

(20) (a) For maximum effective resistance, all the n resistors must be connected in series.

∴ Maximum effective resistance,

$$R_s = nR$$

For min effective resistance, all the n resistors are connected in parallel.

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots + n \text{ terms} = \frac{n}{R}$$

$$R_p = \frac{R}{n}$$

Ratio of max. to min

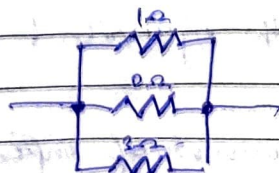
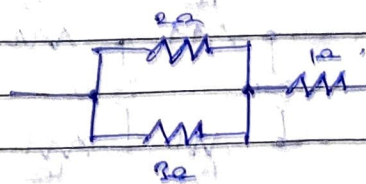
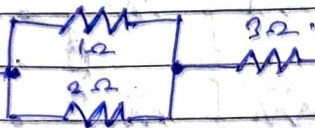
$$\frac{R_s}{R_p} = \frac{nR}{R/n} = \frac{n^2}{1} = n^2 : 1$$

(b) $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$.

$$(i) R = R_p + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$= \frac{1 \times 2}{1 + 2} + 3 = \frac{2}{3} + 3 = \frac{11}{3} \Omega$$

$$(ii) R = \frac{R_2 R_3}{R_2 + R_3} + R_1 = \frac{2 \times 3}{2 + 3} + 1 = \frac{6}{5} + 1 = \frac{11}{5} \Omega$$



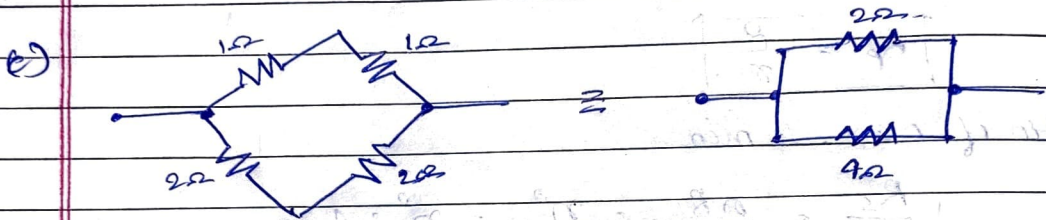
(vii) When the three resistances are connected in series, the equivalent resistance is.

$$R = R_1 + R_2 + R_3 = (1 + 2 + 3) \Omega = 6 \Omega$$

(viii) When all the resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

Equivalent resistance, $R = \frac{6}{11} \Omega$



Resistance R of one such unit

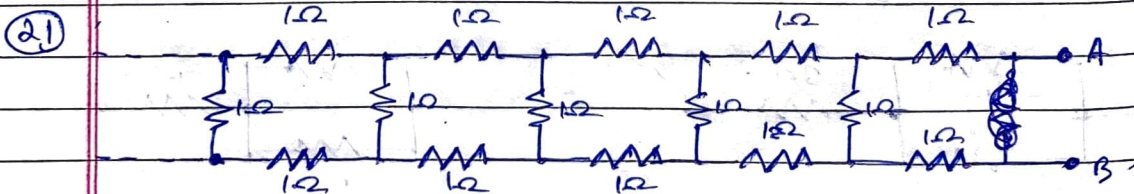
$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$$

or $R = \frac{4}{3} \Omega$

∴ Resistance of the total network (4 such units),

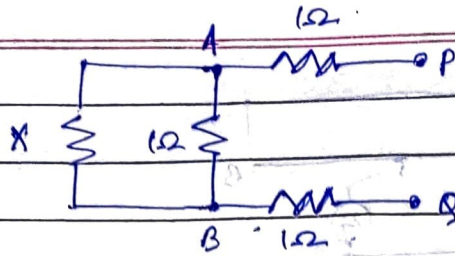
$$= 4 \times \frac{4}{3} = \frac{16}{3} \Omega$$

(x) Equivalent resistance = $5R$.



Let the equivalent resistance of the infinite network be X .

It consists of infinite units of 1Ω , 1Ω and 1Ω resistors.



Resistance between A and B .

= Resistance equivalent to parallel combination of X and 1Ω

$$= \frac{X \times 1}{X + 1} = \frac{X}{X + 1}$$

Resistance between P and Q .

$$= 1 + \frac{X}{X + 1} + 1 = 2 + \frac{X}{X + 1}$$

$$X = 2 + \frac{X}{1 + X}$$

$$X^2 - 2X - 2 = 0$$

$$X = 1 \pm \sqrt{3}$$

$$X = 1 + \sqrt{3} = 2.732\Omega$$

$$\text{Current, } I = \frac{E}{X + 1} = \frac{1.2}{2.732 + 0.5} = 3.713 \text{ A}$$

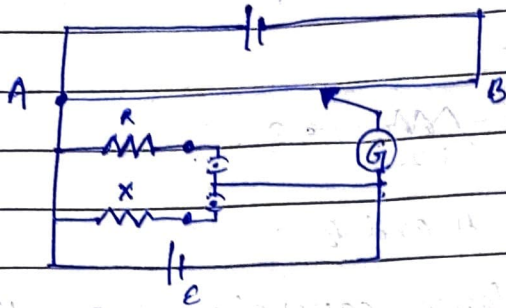
(22) $E_1 = 1.02 \text{ V}$, $I_1 = 67.3 \text{ cm}$

$E_2 = E = ?$, $I_2 = 82.3 \text{ cm}$

$$\frac{E_2}{E_1} = \frac{I_2}{I_1} \therefore \frac{E}{1.02} = \frac{82.3}{67.3}$$

$$E = \frac{82.3}{67.3} \times 1.02 = 1.25 \text{ V}$$

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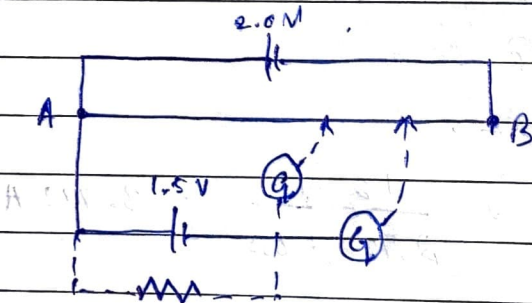
$R = 10.0 \Omega$, $l_1 = 58.3 \text{ cm}$, $X = ?$, $l_2 = 68.5 \text{ cm}$

$$\frac{E_2}{E_1} = \frac{IX}{IR} = \frac{X}{R}$$

$$\frac{E_2}{E_1} = \frac{l_2}{l_1} \quad \therefore \frac{X}{R} = \frac{l_2}{l_1}$$

$$X = \frac{l_2}{l_1} \cdot R = \frac{68.5}{58.3} \times 10 = 11.75 \Omega$$

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$l_1 = 76.3 \text{ cm}$, $l_2 = 64.8 \text{ cm}$, $R = 9.5 \Omega$

$$x = R \left(\frac{l_1 - l_2}{l_2} \right) = 9.5 \left(\frac{76.3 - 64.8}{64.8} \right)$$

$$= \frac{9.5 \times 11.5}{64.8} = 1.7 \Omega$$