(191-112)

HW

Exercise 8.1

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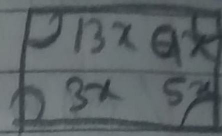
(1) The angles of quadrilateral are in ration 3 : 5 : 9 : 3. Find all the angles of the quadrilateral.



⇒ Let the angles are $3x, 9x, 5x$ & $13x$

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$3x + 5x + 9x + 13x = 360^\circ$

$30x = 360^\circ$

$x = \frac{\overset{12}{\cancel{360}}}{\cancel{30}_{1}}$

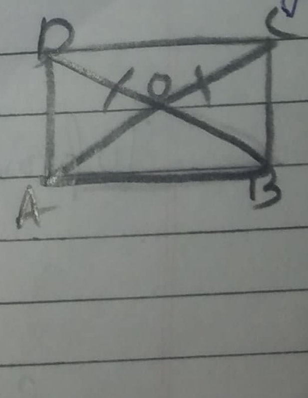
∴ i) $3x \rightarrow 3 \times 12 = 36^\circ$

ii) $5x \rightarrow 5 \times 12 = 60^\circ$

iii) $9x \rightarrow 9 \times 12 = 108^\circ$

iv) $13x \rightarrow 13 \times 12 = 156^\circ$

2) If the diagonals of a parallelogram are equal, then show that it is a rectangle



⇒ Given → ABCD is a llgm.
AC = BD

To prove :- ABCD is a rectangle

proof :- In △DAB and △CBA

AD = BC (opposite sides of llgm)

AB = BA (common)

AC = DB (given) $\angle DAB = \angle CBA$ (CPCT)

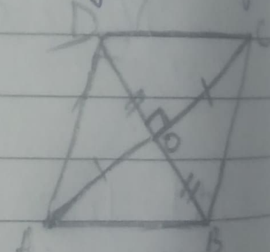
$\Delta DAB \cong \Delta CBA$ (SSS)

$\angle DAB + \angle ABC = 180°$ (Adjacent angles of llgm)

$\angle DAB = \angle ABC = \frac{180}{2}$ 90°

∴ ABCD is a llgm in which ABCD is a rectangle.

3) Show that if the diagonals of a quadri-lateral bisect each other at right angles then it is a rhombus.



⇒ Given → OA = OC
OB = OD
$\angle AOB = \angle BOC$
$\angle COD = \angle DOA$

To prove :- ABCD is llgm,
AB = BC
CD = AD

proof :- In ΔAOB and ΔCOB

OA = OC (given)

$\angle AOB = \angle COB$ (opposite sides of a parallelogram are equal)

$OB = OB$ (common)

$\Delta AOB \cong \Delta COB$ (SAS)

$AB = BC$ (cpct)

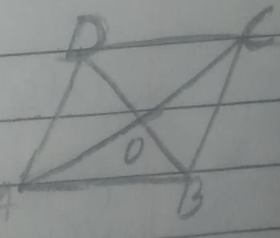
If $AB = BC$

Then $BC = CD$
$CD = AD$
$AD = AB$

opposite sides of a quadrilateral are equal hence ABCD is a parallelogram.

Hence proved that ABCD is a rhombus.

4) Show that the diagonals of a square are equal and bisects each other at right angles.



⇒ Given -: ABCD is a square, AC and BD are diagonal

To prove -: $AC = BD$
$OA = OC$
$OB = OD$
$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

proof :- In ΔDAB = ΔCBA

$\angle DAB = \angle CBA = 90°$

AC = BC (sides of square)

AB = AB (common)

$\Delta DAB \cong CBA$ (SAS)

AC = BD (cpct)

In ΔAOB and ΔCOD

$\angle 1 = \angle 2$ (Alternate angles)

$\angle 3 = \angle 4$ (Alternate angles)

AB = CD (sides of square)

$\Delta AOB \cong \Delta COD$ (ASA)

OA = OC

OB = OD (cpct)

In ΔAOB and ΔCOD

OA = OC (proved)

OB = OB (common)

AB = CB (sides of square)

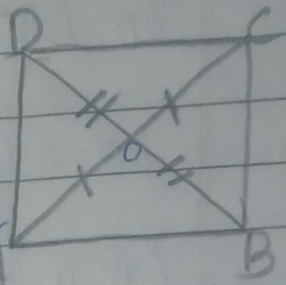
$\Delta AOB \cong \Delta COB$ (SSS)

$\angle AOB = \angle BOC$ (cpct)

$\angle AOB + \angle BOC = 180^\circ$ (linear pair)

Similarly $\angle COD = \angle DOA = 90^\circ$

5) Show that the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



⇒ ~~Given - DO = BO, AO = CO, $\angle DOC = \angle BOA$, $\angle DOA = \angle BOC$~~ ABCD is a quadrilateral and its diagonal AC and BD bisects each other at right angle at O

~~To prove -: ABCD is a square~~
~~AB = BC = CD = DA~~

~~proof -:~~

Given - ABCD is a quadrilateral and its diagonal AC and BD bisects each other at right angle at O.

To prove -: ABCD is a square.

proof -: In ΔAOB and ΔCOD

AO = CO (diagonals bisect each other)

$\angle AOB = \angle COD$ (vertically opposite)

$OB = OD$ (diagonal bisects each other)

$\Delta AOB \cong \Delta COD$ (SAS)

$AB = CD$ (CPCT) — (i)

$\angle OAB = \angle OCD$ (Alternate interior angle)

$\Rightarrow AB \parallel CD$

In ΔAOD and ΔCOD,

$AO = CO$ (Diagonals bisect each other)

$\angle AOD = \angle COD$ (vertically opposite)

$OD = OD$ (Common)

$\Delta AOD \cong \Delta COD$ (SAS)

$\therefore AD = CD$ (CPCT) — (ii)

$\angle ADC = \angle BCD$ (CPCT)

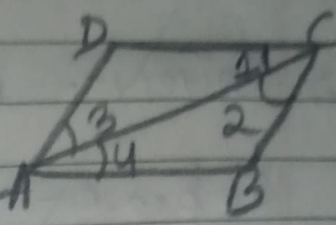
$\angle ADC + \angle BCD = 180^\circ$ (cointerior angles)

$\Rightarrow 2\angle ADC = 180^\circ$

$\angle ADC = 90^\circ$ — (iii)

From (i), (ii), (iii) - ABCD is a square (Hence proved)

6) Diagonal AC of a Parallelogram ABCD bisects ∠A. Show that (i) it bisects ∠C also, ii) ABCD is a rhombus.



⇒ Given :- ABCD is a ||gm
AC bisects ∠A, ∠3 = ∠4

To prove :- ∠1 = ∠2.
ABCD is a rhombus.

proof :- ∠1 = ∠4 (Alternate angle)
∠2 = ∠3

∠3 = ∠4 (given)

∠1 = ∠2

AC bisects AC.

∠2 = ∠3 (Alterate)

∠3 = ∠4 (given)

∠2 = ∠4

In △ABC,

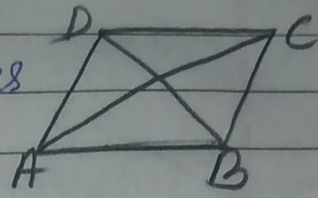
∠2 = ∠4

AB = CB

ABCD is ||gm in which adjacent sides are equal.

∴ ABCD is a rhombus.

7) ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.



→ In Given-: ABCD is a rhombus

$AB = CD = DA = CB$

To prove-: AC bisects ∠A & ∠C

BD bisects ∠B & ∠D

proof -: In △ADC & △ABC

$AB = CD$ (It is a rhombus)

$BC = AD$

$AC = CA$ (common)

$\angle CAD = \angle BAC$

$\angle ACB = \angle ACD$ (Hence, AC bisects ∠A & ∠C)

In △DAB and △BCD

$AB = BC$ (It is a rhombus)

$AD = DC$

$BD = DB$ (common)

$\angle DAB = \angle DCB$

$\angle BCD = \angle BAD$ (Hence BD bisects ∠B & ∠D)

9) In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$.

Show that:

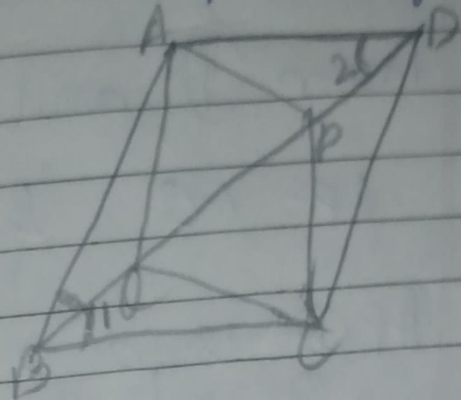
i) $\Delta APD \cong \Delta CQB$

ii) $AP = CQ$

iii) $\Delta AQB \cong \Delta CPD$

iv) $AQ = CP$

v) APCQ is a parallelogram.



⇒ Proof -: In ΔAPD and ΔCQB

$BQ = DP$ (given)

$AD = BC$ (Opposite sides are equal)

$\angle 1 = \angle 2$ (Alternate angles)

$\Delta APD \cong \Delta CQB$ (SAS)

$AP = CQ$ (CPCT) — ①

In ΔAQB & ΔCPD

$AB = DC$ (Opposite sides of ||gm are equal)

BQ = DB (common)

$\angle QDC = \angle ABQ$ (Alternate angles)

$\triangle AQB \cong \triangle CPD$ (SAS)

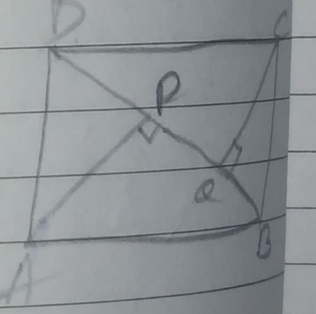
AQ = CP (cpct) —— ②

from ① and ②

AP = CQ & AQ = CP

APCQ is a parallelogram, hence proved.

10) ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that



i) $\triangle APB = \triangle CQD$

ii) AP = CQ.

=) Given-) ABCD is a parallelogram

To prove -) $\triangle APB = \triangle CQD$ & AP = CQ

Proof : In $\triangle APB$ & $\triangle CQD$.

~~AR = CQ (per~~ DC = AB (opp. sides are equal)

$\angle ABP = \angle CDQ$ (Alternate angle)

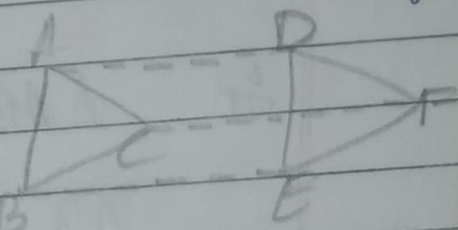
$\angle APB = \angle CQD$ (90° each)

$AB = CD$ (~~ABCD is a llgm~~ opp. side of llgm are equal)

$\Delta APB \cong \Delta CQD$ (AAS)

$AP = CQ$ (cpct)

11) In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively.



=> Proof -: In ΔABC & ΔDEF

$AB = DE$ (given)

$AB \parallel DE$ (given)

ABED is a parallelogram,

hence $BE = AD$, $BE \parallel AD$ (difference between two parallel side are equal). — (1)

$BC = EF$ (given)

$BC \parallel EF$ (given)

BEFC is a parallelogram.

BE = CF, BE ∥ CF (difference b/w two parallel sides are equal) — ②

From ① and ②

AD = CF, AD ∥ CF (difference between two parallel sides are eq)

⇒ ACFD is a parallelogram

AC = DF (Opposite sides of parall-)

In ΔABC and ΔDEF

$$\left.\begin{aligned} AB &= DE \\ BC &= EF \\ AC &= DF \end{aligned}\right\} \begin{aligned} &S \\ &S \\ &S \end{aligned}$$

$\Delta ABC \cong \Delta DEF$ (SSS)