3. State which of the following sets are finite and which infinite are infinite.

i) $A = \{x : x \in Z \text{ and } x < 10\}$

$x < 10 = 9, 8, 7, 6, 5, \ldots\ldots$

$\therefore A = \{x : x \in Z \text{ and } x < 10\}$ is infinite

ii) $B = \{x : x \in W \text{ and } 5x - 3 \leq 20\}$

$5x - 3 \leq 20$

Add 3 to both sides

$5x - 3 + 3 \leq 20 + 3$

$5x \leq 23$

$x \leq \frac{23}{5} = 4.6$

$= 0, 1, 2, 3, 4$

$\therefore B = \{x : x \in W \text{ and } 5x - 3 \leq 20\}$ is finite.

iii) $P = \{y : y = 3x - 2, x \in N \text{ and } x \geq 5\}$

$= x \geq 5 = 6, 7, 8, 9, \ldots\ldots$

$y = 3x - 2$

$= 3 \times 6 - 2 = 18 - 2 = 16$

$y = 3x - 2$

$= 3 \times 7 - 2 = 21 - 2 = 19$

$y = 3x - 2$

$= 3 \times 8 - 2 = 24 - 2 = 22$

$y = 3x - 2$

$= 3 \times 9 - 2 = 27 - 2 = 25$

$xy = \{16, 19, 22, 25 \ldots\ldots\}$

So the given set is infinite.

3(iii) $M = \{r : r = \frac{3}{n} ; n \in W \text{ and } 6 < n \leq 15\}$

$n = \{7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$r = \left\{\frac{3}{7}, \frac{3}{8}, \frac{3}{9}, \frac{3}{10}, \frac{3}{11}, \frac{3}{12}, \frac{3}{13}, \frac{3}{14}, \frac{3}{15}\right\}$

So the given set is finite.

4) Find, which of the following sets are singleton sets:

(i) The set of points of intersection of two non-parallel straight lines on the same plane.

It is a single-ton sets

(ii) $A = \{x : 7x - 3 = 11\}$

$\Rightarrow 7x - 3 = 11$

$7x = 11 + 3 = 14$

$x = \frac{14}{7} = 2$

$\therefore$ Given set is singleton set.

(iii) $B = (y : 2y + 1 < 3 \text{ and } y \in W)$

$2y + 1 < 3$

Subtracting 1 to both sides

$= 2y + 1 - 1 < 3 - 1$

$2y < 2$

Dividing 2 to both sides

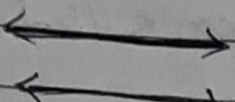
$= \frac{2y}{2} < \frac{2}{2} 1$

$= \therefore y < 1 = \{0\}$

$\therefore$ Given set is singleton.

5. Find which of the following sets are empty.

(i) The set of points of intersection of two parallel lines.

Ans 

∴ Given set is are empty.

(ii) $A = \{x : x \in N \text{ and } 5 < x \leq 6\}$

• Since 6 is natural number.

∴ Given set is not empty.

(iii) $B = \{x : x^2 + 4 = 0 \text{ and } x \in N\}$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \sqrt{-4}$

∴ Given set are empty.

(iv) $C = \{\text{even number between 6 and 10}\}$

$C = \{8, 10\}$

∴ Given set are not empty.

(v) $D = \{\text{prime numbers between 7 and 11}\}$

• Since there are no prime numbers between 7 and 11

∴ Given set are empty.

6. (i) Are the sets $A = \{4, 5, 6\}$ and $B = \{x : x^2 - 5x - 6 = 0\}$ disjoint?

Ans $A = \{4, 5, 6\}$

$B = x^2 - 5x - 6 = 0$

$= x^2 - (6x - x) - 6 = 0$

$= x^2 - x(6 - 1) - 6 = 6$

~~$= x(x) - 6$~~

$= x(x - 6) + 1(x - 6) = 6$

$= (x - 6)(x + 1) = 6$

$= x - 6 = 0, \quad x + 1 = 0$

$x = 6 \qquad x = -1$

∴ $B = \{6, -1\}$

∴ No, the sets A and B are not disjoint because 1 element is common which is 6.

iii) Are the sets $A = \{b, c, d, e\}$ and $B = \{x : x$ is a letter in a word 'MASTER'$\}$ joint.

$A = \{b, c, d, e\}$

$B = \{m, a, s, t, e, r\}$

Yes, the set A and B are joint as element "e" is common in both the sets.

4) State whether the following pair of sets are equivalent or not.

i) $A = \{x : x \in N$ and $11 \geq 2x - 1\}$ and $B = \{y : y \in W$ and $3 \leq y \leq 9\}$

Ans) $11 \geq 2x - 1$

Adding 1 to both sides

$= 1 + 11 \geq 2x - 1 + 1$

$= 12 \geq 2x$

$= \frac{12}{2} \geq x$

$\therefore 6 \geq x$

$B = 3 \leq y \leq 9$

$\{3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4, 5, 6\}$

∴ Set A and B are not equivalent

ii) Set of integers and set of natural numbers

Ans) Both the sets must contain same number of element.

∴ Given set are equivalent.

iii) Set of whole numbers and set of multiples of 3

Ans) Whole number = 0, 1, 2, 3, 4, 5, 6, ...

Multiple of 3 = 3, 6, 9, 12, 15, ...

Both the sets must contain same number of element.

∴ Given set are equivalent.

iv) $P = \{5, 6, 7, 8\}$ and $M = \{x : x \in W \text{ and } x < 4\}$

$P = \{5, 6, 7, 8\}$

$M = \{x < 4\}$

$= \{0, 1, 2, 3, 4\}$

Both the sets doesnot contain equal element

∴ Sets P and M are not equivalent.

v) $A = \{2, 4, 6, 8\}$

$B = \{2n : n \in N \text{ and } n < 5\}$

$= n < 5$

$n = 1, 2, 3, 4$

$2n = 2 \times 1 = 2$

$2 \times 2 = 4$

$2 \times 3 = 6$

$2 \times 4 = 8$

∴ $A = \{2, 4, 6, 8\}$

$B = \{2, 4, 6, 8\}$

∴ Given sets are equal.

iv) $M = \{x : x \in W \text{ and } x + 3 < 8\}$ and

$N = \{y : y = 2n - 1, n \in N \text{ and } n < 3\}$

$M = x + 3 < 8$

Subtracting 3 both sides

$x + 3 - 3 < 8 - 3$

$x < 5$

$= \{0, 1, 2, 3, 4\}$

$N = n < 5$

$= 1, 2, 3, 4$

$y = 2n - 1$

$= 2 \times 1 - 1 = 1$

$= 2 \times 2 - 1 = 3$

$= 2 \times 3 - 1 = 5$

$= 2 \times 4 - 1 = 7$

∴ Given set $N = \{1, 3, 5, 7\}$

∴ Given sets are not equal.

iii) $E = \{x : x^2 + 8x - 9 = 0\}$ and $F = 1, -9$

$E = x^2 + 8x - 9 = 0$

$= x^2 + 9x - x - 9 = 0$

$= x(x+9) - 1(x+9) = 0$

$(x+9)(x-1) = 0$

$x+9 = 0, \quad x-1 = 0$

$x = -9 \qquad x = 1$

$x = \{-9, 1\}$

$F = \{1, -9\}$

$\therefore E = \{1, -9\}, \quad F = \{1, -9\}$

∴ Set E and F have same element

∴ Set E and F are equal.

iv) $A = \{x : x \in N, x < 3\}$ and $B = \{y : y^2 - 3y + 2 = 0\}$

$A = x < 3$

$= \{1, 2\}$

$B = y^2 - 3y + 2 = 0$

$= y^2 - 2y - y + 2 = 0$

$= y(y-2) - 1(y+2) = 0$

$= (y-2)(y-1) = 0$

$y - 2 = 0, \quad y - 1 = 0$

$y = 2 \qquad y = 1$

$\therefore A = \{1, 2\}, \quad B = \{1, 2\}$

∴ Set A and B have same element.

∴ Set A and B are equal.

q) i) The set of multiples of 8

Ans) Multiples of 8 = 8, 16, 24, 32, ...

So the set is infinite.

ii) The set of intergers less that 10

Ans) ....., 6, 7, 8, 9

∴ Given set is infinite.

iii) The set of whole numbers less than 12.

Ans) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

∴ Given set is finite.

iv) $\{x : x = 3n - 2, n \in W, n \leq 8\}$

Whole number $= \{0, 1, 2, 3, 4, 5, 6, 7\}$

$x = 3n - 2$

$x = 3 \times 0 - 2 = -2$

$3 \times 1 - 2 = 1$

$3 \times 2 - 2 = 4$

$3 \times 3 - 2 = 7$

$3 \times 4 - 2 = 10$

$3 \times 5 - 2 = 13$

$3 \times 6 - 2 = 16$

$3 \times 7 - 2 = 19$

$\therefore x = \{-2, 1, 4, 7, 10, 13, 16, 19\}$

∴ Given set is finite.

v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

$Z = n \leq 8$

$= \{\ldots\ldots 5, 6, 7, 8\}$

$x = 3n - 2$

$= 3 \times 5 - 2 = 13$

$= 3 \times 6 - 2 = 16$

$3 \times 7 - 2 = 19$

$3 \times 8 - 2 = 22$

$Z = \{\ldots\ldots 5, 6, 7, 8\}$

$x = \{\ldots\ldots 13, 16, 19, 22\}$

∴ Given set is infinite

vi) $x : x = \frac{n - 2}{n + 1}, n \in W$

Whole number = 1, 2, 3, 4, 5, ...

$x = \frac{n-2}{n+1}$

$= \frac{1-2}{1+1} = \frac{-1}{2}$

$= \frac{2-2}{2+1} = \frac{0}{3}$

$= \frac{3-2}{3+1} = \frac{1}{4}$

$= \frac{4-2}{4+1} = \frac{2}{5}$

$= \frac{5-2}{5+1} = \frac{3}{6}$, so on ..

∴ Given set is infinite

(iii) The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are equivalent sets.

Ans: 2, 4, 8, 10, ..., 20 are even natural number

1, 3, 5, 7, ..., 19 are odd natural number.

∴ The given statement is true.

(ii) If E = {factors of 16} and F = {factors of 20}, then E = F

E = {1, 2, 4, 8, 16}

F = {1, 2, 4, 5, 10, 20}

∴ Given statement is false.

(iii) The set A = {integers less than 20} is a finite set.

A = {....., 18, 19, 20}

∴ Given statement is false

(iv) If A = {x : x is an even prime number.} then set A is empty.

Ans: A = {2} 2 is even prime number

∴ Given statement is false.

vi) The set of odd prime numbers is the empty set.
Odd prime numbers = {3, 5, 7, 11, 13, ...}
∴ Given statement is false.

vii) The set of squares of integers and the same of whole numbers are equal sets.
{0, ±2, ±1, ±3, ±4, ±5, ...}
= {0, +4, +1, +9, +16, +25, ...}
Whole numbers = {0, 1, 2, 3, 4, 5 .....}
∴ Given statement is false.

viii) If n(P) = n(M) then P ↔ M.
n(m) = n(p)
No. of element P = No. of element M
Set P = Set M
∴ Given statement is true.

viii) If set P = set M, then n(P) = n(M)
n(m) = n(p)
No. of element P = No. of element M
Set P = Set M
∴ Given statement is True

ix) n(A) = n(B) ⇏ A = B
⇏ A = 1, 2, 3, 4 , B = 5, 6 7 8
Both are equivalent but not equal.