CUBES AND CUBE-ROOTS

Exercise-4(A)

(1) Find the cube of :

i) 7

→ $(7)^3 = 7 \times 7 \times 7 = 343$

ii) 11

→ $(11)^3 = 11 \times 11 \times 11 = 1331$

iii) 16

→ $(16)^3 = 16 \times 16 \times 16 = 4096$

iv) 23

→ $(23)^3 = 23 \times 23 \times 23 = 12167$

v) 31

→ $(31)^3 = 31 \times 31 \times 31 = 29791$

vi) 42

→ $(42)^3 = 42 \times 42 \times 42 = 74088$

vii) 54

→ $(54)^3 = 54 \times 54 \times 54 = 157464$

② Find which of the following are perfect cubes:

i) 243

→

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$243 = 3 \times 3 \times 3 \times 3$

$= (3 \times 3 \times 3) \times 3$

$= 3^3 \times 3$

∴ 297 is not a perfect cube.

ii) 588

$$\begin{array}{r|l} 2 & 588 \\ \hline 2 & 294 \\ \hline 7 & 147 \\ \hline 7 & 21 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$588 = 2 \times 2 \times 7 \times 7 \times 3$

∴ 588 is not a perfect cube

iii) 1331

```
11 | 1331
11 | 121
11 | 11
   | 1
```

$1331 = 11 \times 11 \times 11 = (11)^3$

∴ 131 is a perfect cube

iv) 24000

```
2 | 24000
2 | 12000
2 | 6000
2 | 3000
2 | 1500
2 | 750
3 | 375
5 | 125
5 | 25
5 | 5
  | 1
```

$24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$
$= (2)^3 \times (2)^3 \times (5)^3 \times 3$

∴ 24000 is not a perfect cube

vi) 1728

$$
\begin{array}{r|l}
2 & 1728 \\
\hline
2 & 864 \\
\hline
2 & 432 \\
\hline
2 & 216 \\
\hline
2 & 108 \\
\hline
2 & 54 \\
\hline
3 & 27 \\
\hline
3 & 9 \\
\hline
3 & 3 \\
\hline
 & 1
\end{array}
$$

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2)^3 \times (2)^3 \times (3)^3$

$\therefore$ 1728 is a perfect cube.

vii) 1938

$$
\begin{array}{r|l}
2 & 1938 \\
\hline
3 & 969 \\
\hline
17 & 323 \\
\hline
19 & 19 \\
\hline
 & 1
\end{array}
$$

$1938 = 2 \times 3 \times 17 \times 19$

$\therefore$ 1938 is not a perfect cube

(3) Find the cubes of :

i) 2·1

→ $(2·1)^3 = 2·1 \times 2·1 \times 2·1 = 9·261$

ii) 0·4

→ $(0·4)^3 = 0·4 \times 0·4 \times 0·4 = 0·064$

iii) 1·6

→ $(1·6)^3 = 1·6 \times 1·6 \times 1·6 = 4·096$

iv) 2·5

→ $(2·5)^3 = 2·5 \times 2·5 \times 2·5 = 15·625$

v) 0·12

→ $(0·12)^3 = 0·12 \times 0·12 \times 0·12 = 0·001728$

vi) 0·02

→ $(0·02)^3 = 0·02 \times 0·02 \times 0·02 = 0·000008$

vii) 0·8

→ $(0·8)^3 = 0·8 \times 0·8 \times 0·8 = 0·512$

(4) Find the cubes of :

i) $\frac{3}{7} = \left(\frac{3}{7}\right)^3$

$= \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = \frac{24}{343}$

ii) $\frac{8}{9}$

$= \left(\frac{8}{9}\right)^3 = \frac{8 \times 8 \times 8}{9 \times 9 \times 9} = \frac{512}{729}$

iii) $\frac{10}{13}$

$= \left(\frac{10}{13}\right)^3 = \frac{10 \times 10 \times 10}{13 \times 13 \times 13} = \frac{1000}{2197}$

iv) $1\frac{2}{7}$

$= \left(1\frac{2}{7}\right)^3 = \left(\frac{1 \times 7 + 2}{7}\right)^3 = \left(\frac{9}{7}\right)^3$

$= \frac{9 \times 9 \times 9}{7 \times 7 \times 7} = \frac{729}{343} = 2\frac{43}{343}$

v) $2\frac{1}{2}$

$= \left(2\frac{1}{2}\right)^3 = \left(\frac{5}{2}\right)^3$

$= \frac{5 \times 5 \times 5}{2 \times 2 \times 2} = \frac{125}{8} = 15\frac{5}{8}$

(5) Find the cubes of :

i) -3

$\rightarrow (-3)^3 = -3 \times -3 \times -3$

$= -(3 \times 3 \times 3) = -27$

ii) -7

$\rightarrow (-7)^3 = -7 \times -7 \times -7$

$= -343$

iii) -12

$\rightarrow (-12)^3 = -12 \times -12 \times -12$

$= -1728$

iv) -18

$\rightarrow (-18)^3 = -18 \times -18 \times -18$

$= -5832$

v) -25

$\rightarrow (-25)^3 = -25 \times -25 \times -25$

$= -15625$

vi) -30

$\rightarrow (-30)^3 = -30 \times -30 \times -30$

$= -27000$

(6) Which of the following are cubes of :

i) an even number

→ 216, 8000, 4096

ii) an odd number

→ 729, 3375, 125, 343, 9261

(7) Find the least numbere by which 1323 must be multiplied so that the product is a perfect cube.

→

$$\begin{array}{r|l} 3 & 1323 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

= The preime factore of 1323 are =
$3 \times 3 \times 3 \times 7 \times 7$
$= (3 \times 3 \times 3) \times 7 \times 7$

Clearly, 1323 must be multiplied by 7

(8) Find the smallest numbere by which 8768 must be divided so that the quotient is a perefect cube.

→

$$\begin{array}{r|l} 2 & 8768 \\ \hline 2 & 4384 \\ \hline 2 & 2192 \\ \hline 2 & 1096 \\ \hline 2 & 548 \\ \hline 2 & 274 \\ \hline 137 & 137 \\ \hline & 1 \end{array}$$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 137$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 137$

clearly, 8678 must be divided by 137

(9) Find the smallest number by which 27783 be multiplied to get a perfect cube number

$$
\begin{array}{r|l}
3 & 27783 \\
3 & 9261 \\
3 & 3087 \\
3 & 1029 \\
7 & 343 \\
7 & 49 \\
7 & 7 \\
 & 1
\end{array}
$$

$= 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (3 \times 3 \times 3) \times (7 \times 7 \times 7) \times 3$

Clearly, 27783 must be multiplied by $3 \times 3 = 9$

(10) With what least number must 8640 be divided so that the quotient is a perfect cube ?

→

2 | 8640
2 | 4320
2 | 2160
2 | 1080
2 | 540
2 | 270
3 | 135
3 | 45
3 | 15
5 | 5
1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5$

Clearly, 8640 must be divided by 5

(11) Which is the smallest number that must be multiplied to 77175 to make it a perfect cube?

→

3 | 77175
3 | 25725
3 | 8575
5 | 1715
7 | 343
7 | 49
7 | 7
1

$= 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 7$

$= (7 \times 7 \times 7) \times 3 \times 3 \times 5 \times 5$

Clearly, 7 7 1 7 5 must be multiplied by $3 \times 5 = 15$

Exercise - 4(B)

(1) Find the cube-roots of :

i) 64

$= \sqrt[3]{64}$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$\Rightarrow \sqrt[3]{64} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2$

$= 4$

ii) $343 = \sqrt[3]{343}$

7	343
7	49
7	7
	1

$\sqrt[3]{343} = (7 \times 7 \times 7)$

$= 7$

iii) 729

$$
\begin{array}{r|l}
3 & 729 \\
\hline
3 & 243 \\
\hline
3 & 81 \\
\hline
3 & 27 \\
\hline
3 & 9 \\
\hline
3 & 3 \\
\hline
 & 1
\end{array}
$$

$\sqrt[3]{729} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$= 3 \times 3 = 9$

iv) 1728

$$
\begin{array}{r|l}
2 & 1728 \\
\hline
2 & 864 \\
\hline
2 & 432 \\
\hline
2 & 216 \\
\hline
2 & 108 \\
\hline
2 & 54 \\
\hline
3 & 27 \\
\hline
3 & 9 \\
\hline
3 & 3 \\
\hline
 & 1
\end{array}
$$

$\sqrt[3]{1728} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$= 2 \times 2 \times 3$

$= 12$

v) 9261

$$\begin{array}{r|l} 3 & 9261 \\ \hline 3 & 3087 \\ \hline 3 & 1029 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\sqrt[3]{9261} = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$= 3 \times 7 = 21$

vi) 4096

$$\begin{array}{r|l} 2 & 4096 \\ \hline 2 & 2048 \\ \hline 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$\sqrt[3]{4096} = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 = 16$

vii) 8000

$$\begin{array}{r|l} 4 & 8000 \\ 4 & 2000 \\ 4 & 500 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$\sqrt[3]{8000} = (4\times4\times4)\times(5\times5\times5)$

$= 4\times5 = 20$

viii) 3375

$$\begin{array}{r|l} 5 & 3375 \\ 5 & 675 \\ 5 & 135 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$\sqrt[3]{3375} = (5\times5\times5)\times(3\times3\times3)$

$= 5\times3 = 15$

(2) Find the cube-roots of :

i) $\frac{27}{64} = \sqrt[3]{\frac{27}{64}} = \frac{\sqrt{3\times3\times3}}{\sqrt{4\times4\times4}} = \frac{3}{4}$

ii) $\frac{125}{216} = \frac{\sqrt{125}}{\sqrt{216}} = \frac{\sqrt{5\times5\times5}}{\sqrt{6\times6\times6}} = \frac{5}{6}$

iii) $\frac{343}{512} = \frac{\sqrt[3]{343}}{\sqrt[3]{512}} = \frac{\sqrt{7\times7\times7}}{\sqrt{8\times8\times8}} = \frac{7}{8}$

iv) $64 \times 729 = \sqrt[3]{64\times729}$

$= \sqrt{4\times4\times4\times9\times9\times9} = 4\times9$

$= 36$

v) 64×27

$= \sqrt[3]{64\times27}$

$= \sqrt{4\times4\times4\times3\times3\times3} = 4\times3$

$= 12$

vi) $729 \times 8000 = \sqrt[3]{729\times8000}$

$= \sqrt{9\times9\times9\times20\times20\times20}$

$= 9\times20$

$= 180$

vii) 3375×512

$= \sqrt[3]{3375\times512}$

$= \sqrt{15\times15\times15\times8\times8\times8}$

$= 15\times8$

$= 120$

(3) Find the cube-roots of :

i) -216

$\rightarrow \sqrt[3]{-216} = \sqrt{-6 \times -6 \times -6} = -6$

ii) -512

$\rightarrow \sqrt[3]{-512} = \sqrt{-8 \times -8 \times -8} = -8$

iii) -1331

$\rightarrow \sqrt[3]{-1331} = \sqrt{-11 \times -11 \times -11} = -11$

iv) $-\frac{27}{125} = \frac{\sqrt{27}}{\sqrt{125}} = -\sqrt{\frac{3 \times 3 \times 3}{5 \times 5 \times 5}} = -\frac{3}{5}$

v) $\frac{-64}{343} = \frac{\sqrt[3]{-64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{-4 \times -4 \times -4}}{\sqrt[3]{7 \times 7 \times 7}} = \frac{-4}{7}$

vi) $-\frac{512}{343} = -\sqrt[3]{\frac{512}{343}} = \frac{-\sqrt[3]{8 \times 8 \times 8}}{\sqrt[3]{7 \times 7 \times 7}} = -\frac{8}{7}$

vii) $-2197 = \sqrt[3]{-2197}$

$13 \mid 2197$

$13 \mid 169$

$13 \mid 13$

1

$= \sqrt[3]{-13 \times -13 \times -13}$

$= -13$

viii) $-5832 = \sqrt[3]{-5832}$

2 | 5832
2 | 2916
2 | 1458
3 | 729
3 | 243
3 | 81
3 | 27
3 | 9
3 | 3
1

$= \sqrt{-2x-2x-2x-3x-3x-3x-3x-3x-3}$
$= -2x-3\ x-3 = -18$

ix) -2744000
$= \sqrt[3]{-2744000}$

2 | 2744000
2 | 1372000
2 | 686000
7 | 343000
7 | 49000
7 | 7000
10 | 1000
10 | 100
10 | 10
1

$= \sqrt{-2x-2x-2x-7x-7x-7x-10x-10x-10}$
$= -2x-7x-10$
$= -140$

(4) Find the cube-root of :

i) 2.744

$$= \sqrt[3]{\frac{2744}{1000}}$$

2 | 2744
2 | 1372
2 | 686
7 | 343
7 | 49
7 | 7
1

$$= \sqrt[3]{\frac{2\times2\times2\times7\times7\times7}{10\times10\times10}}$$

$$= \frac{2\times7}{10} = \frac{14}{10} = 1.4$$

ii) 9.261

$$= \sqrt[3]{\frac{9261}{1000}} = \sqrt{\frac{3\times3\times3\times7\times7\times7}{10\times10\times10}}$$

$$= \frac{3\times7}{10} = \frac{21}{10} = 2.1$$

iii) $0.000027 = \sqrt[3]{\frac{27}{1000000}}$

$= \sqrt[3]{\frac{3 \times 3 \times 3}{100 \times 100 \times 100}} = \frac{3}{100} = 0.03$

iv) $-0.512 = \sqrt[3]{\frac{-512}{1000}} = \sqrt{\frac{-8 \times -8 \times -8}{10 \times 10 \times 10}}$

$= \frac{-8}{10} = -0.8$

v) -15.625

$= \sqrt[3]{\frac{-15625}{1000}}$

5 | 15625
5 | 3125
5 | 625
5 | 125
5 | 25
5 | 5
1

$= \sqrt{\frac{-(5 \times 5 \times 5) \times (5 \times 5 \times 5)}{10 \times 10 \times 10}}$

$= \frac{-5 \times 5}{10} = \frac{-25}{10} = -2.5$

vii) $-125 \times 1000 = \sqrt{-125 \times 100}$

$= \sqrt{-(5 \times 5 \times 5) \times (10 \times 10 \times 10)}$

$= -5 \times 10 = -50$

(5) Find the smallest number by which 26244 should be divided so that the quotient is a perfect cube

→

```
2 | 26244
2 | 13122
3 | 6561
3 | 2187
3 | 729
3 | 243
3 | 81
3 | 27
3 | 9
3 | 3
    1
```

= 2 X 2 X 3 X 3 X 3 X 3 X 3 X 3 X 3 X 3
= (3 X 3 X 3) X (3 X 3 X 3) X 3 X 3 X 2 X 2

Hence 26244 must be divided by 3 X 3 X 2 X 2 = 36

(6) What is least number by which 30375 should be multiplied to get a perfect cube?

→

```
3 | 30375
3 | 10125
3 | 3375
3 | 1125
3 | 375
5 | 125
5 | 25
5 | 5
    1
```

= 3 X 3 X 3 X 3 X 3 X 5 X 5 X 5
= (3 X 3 X 3) X (5 X 5 X 5) X 3 X 3

Hence, 30375 must be multiplied with 3.

(7) Find the cube-roots of:

i) 700 X 2 X 49 X 5

```
2 | 700
2 | 350
5 | 175
5 | 35
7 | 7
    1
```

= 2 X 2 X 5 X 5 X 7 X 2 X 7 X 7 X 5
= (2 X 2 X 2) X (5 X 5 X 5) X (7 X 7 X 7)
= 2 X 5 X 10 = 70

ii) -216×1728

2 | 216
2 | 108
2 | 54
3 | 27
3 | 9
3 | 3
1

2 | 1728
2 | 864
2 | 432
2 | 216
2 | 108
2 | 54
3 | 27
3 | 9
3 | 3
1

$= -(2\times2\times2\times3\times3\times3) \times (2\times2\times2\times2\times2\times2\times3\times3\times3)$

$= -2\times3\times2\times2\times3$

$= -72$

iii) -64×-125

$= -(4\times4\times4) \times -(5\times5\times5)$

$= -4 \times -5 = 20$

iv) $\frac{-27}{343} = \frac{3\times3\times3}{7\times7\times7} = -\frac{3}{7}$

v) $\frac{729}{-1331} = \frac{(9\times9\times9)}{-(11\times11\times11)} = -\frac{9}{11}$

vi) $250.047 = \frac{250047}{1000}$

3 | 250047
3 | 83349
3 | 27783
3 | 9261
3 | 3087
3 | 1029
7 | 343
7 | 49
7 | 7
1

$= \frac{(3\times3\times3) \times (3\times3\times3) \times (7\times7\times7)}{(10\times10\times10)}$

$= \frac{3\times3\times7}{10} = \frac{63}{10} = 6.3$

vii) -175616

2 | 175616
2 | 87808
2 | 43904
2 | 21952
2 | 10976
2 | 5488
2 | 2744
1372

vii) -175616

$$
\begin{array}{r|l}
2 & 175616 \\
\hline
2 & 87808 \\
\hline
2 & 43904 \\
\hline
2 & 21952 \\
\hline
2 & 10976 \\
\hline
2 & 5488 \\
\hline
2 & 2744 \\
\hline
2 & 1372 \\
\hline
2 & 686 \\
\hline
7 & 343 \\
\hline
7 & 49 \\
\hline
7 & 7 \\
\hline
 & 1
\end{array}
$$

$$= -[(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(7\times7\times7)]$$

$$= -[2\times2\times2\times7] = -56$$