Exercise 4.5.

Q1. (i) $x^2 - 7x + 3 = 0$.

$\Rightarrow 2\left\{x^2 - \frac{7}{2}x + \frac{3}{2}\right\} = 0$.

$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$.

$\Rightarrow (x)^2 - \left(\frac{7}{2}x\right) + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$.

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49}{16} - \frac{3}{2}\right) = 0$.

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49 - 24}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{25}{16}\right) = 0$.

$\Rightarrow \left(x - \frac{7}{4} + \frac{5}{4}\right)\left(x - \frac{7}{4} - \frac{5}{4}\right) = 0$.

$\Rightarrow \left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right) = 0$.

$\Rightarrow (x - 3)\left(x - \frac{1}{2}\right) = 0$.

$\Rightarrow x = 3$ or $x = \frac{1}{2}$.

$\Rightarrow \therefore$ the required roots are $\frac{1}{2}$ & 3.

(ii) $2x^2 + x - 4 = 0$.

$\Rightarrow x^2 + \frac{x}{2} - 2 = 0$.

$\Rightarrow x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$.

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$.

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \left(\frac{33}{16}\right) = 0$.

$\Rightarrow \left(x + \frac{1}{4} + \frac{\sqrt{33}}{4}\right)\left(x + \frac{1}{4} - \frac{\sqrt{33}}{4}\right) = 0$.

$\left(x + \frac{1 + \sqrt{33}}{4}\right)\left(x + \frac{1 - \sqrt{33}}{4}\right) = 0$.

$\Rightarrow x = \frac{-1 - \sqrt{33}}{4}$ or $x = \frac{-1 + \sqrt{33}}{4}$.

Q1 · (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$.

$\rightarrow (2x + \sqrt{3})^2 = 0$.

$\rightarrow (2x + \sqrt{3})(2x + \sqrt{3}) = 0$.

$2x + \sqrt{3} = 0$.

$\rightarrow x = \frac{-\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$.

$\rightarrow x^2 + \frac{x}{2} + 2 = 0$

$\rightarrow x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$.

$\rightarrow \left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{16} - 2\right) = 0$.

$\rightarrow \left(x + \frac{1}{4}\right)^2 - \left(\frac{-31}{16}\right) = 0$.

$\rightarrow \left(x + \frac{1}{4}\right)^2 - \left(\frac{\sqrt{(-31)}}{4}\right)^2 = 0$.

$\therefore$ roots do not exist.

Q2 · (i) $2x^2 - 7x + 3 = 0$.

$ax^2 + bx + c = 0$.

$a = 2, b = -7$ & $c = 3$.

$D = b^2 - 4ac$.

$= (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$.

$D > 0$.

Let roots are α & β.

$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-7) + \sqrt{25}}{2 \times 2}$

$= \frac{7 + 5}{4} = \frac{12}{4} = 3$.

$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-7) - \sqrt{25}}{2 \times 2} = \frac{7 - 5}{4}$

$= \frac{2}{4} = \frac{1}{2}$ $\therefore$ roots are $3, \frac{1}{2}$.

(ii) $2x^2 - x + 4 = 0$.

this is of form $ax^2 + bx + c = 0$.

$a = 2, b = 1$ & $c = -4$.

$D = b^2 - 4ac$.

$= (1)^2 - 4 \times 2 (-4) = 1 + 32 = 33$.

$D > 0$

Let roots are α & β.

$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-1 + \sqrt{33}}{4}$.

$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-1 - \sqrt{33}}{4}$.

$\therefore$ the roots are $\frac{-1 + \sqrt{33}}{4} - \frac{1 - \sqrt{33}}{4}$.

(iii) $2x^2 - x + 4 = 0$.

where $a = 2$.

$b = 1$.

$c = 4$.

$D = b^2 - 4ac$.

$= (1)^2 - 4 \times 2 \times 4 = 1 - 32 =$

$31 < 0$.

$\therefore$ Roots do not exist as D is negative.

Q3. i) $x - \frac{1}{x} = 3$.

x, we get:

$x^2 - 1 = 3x$.

$\Rightarrow x^2 - 3x - 1 = 0$.

This is a quadratic equation

$a = 1$.

$b = -3$.

$c = -1$.

$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$.

$= \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$.

$\Rightarrow x = \frac{3+\sqrt{13}}{2}$.

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$,

$x \neq -4, 7$.

$(x-7) - (x+4) = \frac{11}{30} \times (x+4)(x-7)$.

$\Rightarrow x - 7 - x - 4 = \frac{11}{30}$.

$(x^2 - 4x - 7x - 28)$.

$\Rightarrow -11 = \frac{11}{30}(x^2 - 3x - 28)$.

$\Rightarrow x^2 - 3x - 28 = -30$.

$\Rightarrow x^2 - 3x + 2 = 0$.

Quadratic equation,

$a = 1$.

$b = -3$.

$c = 2$.

$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$.

$= \frac{3 \pm \sqrt{9-8}}{2}$.

$\Rightarrow \frac{3 \pm \sqrt{2}}{2} = \frac{3 \pm 1}{2}$.

Either $x = \frac{3+1}{2}$ or $x = \frac{3-1}{2}$.

$\Rightarrow x = 2$ or $x = 1$.

Q4. Let the present age of Rehman be x years.

3 years ago Rehman's age was $= (x-3)$ years.

5 years from now Rehman's age will be $= (x+5)$ years.

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$.

$\Rightarrow 6x + 6 = x^2 - 2x - 15$. $\Rightarrow x^2 - 4x - 21 = 0$.

$\Rightarrow x^2 - 7x + 3x - 21 = 0$. $\Rightarrow x(x-7) + 3(x-7) = 0$.

$\Rightarrow (x+3)(x-7) = 0$.

$\Rightarrow x + 3 = 0$ or $x - 7 = 0$. $\therefore$ present age of Rehman is 7 years.

$\Rightarrow x = 7$ or -3.

05. Shefali's marks in Mathematics be x.

Then, her marks in eng. will be 30 - x.

$(x+2)(30-x-3) = 210$.

$\Rightarrow (x+2)(27-x) = 210$.

$\Rightarrow -x^2 + 25x + 54 = 210$.

$\Rightarrow x^2 - 25x + 156 = 0$.

$\Rightarrow (x-12)(x-13) = 0$.

$\Rightarrow x = 12$.

If $x = 12$, then marks in Maths = 12.

English = 30 - 12 = 18.

If $x = 13$, Mark in Maths = 13.

English = 30 - 13 = 17.

07. Let the smaller no. = x.

larger no. will be $\frac{x^2}{8}$.

$\left(\frac{x^2}{8}\right)^2 - x^2 = 180$.

$\Rightarrow x^4 - 64x^2 - 11520 = 0$.

$y^2 - 64y - 11520 = 0$.

$\Rightarrow$ here, $a = 1$.

$\Rightarrow b = -64$.

$c = -11520$.

$\therefore D = b^2 - 4ac$.

$= (-64)^2 - 4 \times 1 \times (-11520)$.

$= 4096 + 46080 = 50176$.

$\therefore y = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-64) \pm \sqrt{50176}}{2 \times 1} = \frac{64 \pm 224}{2}$.

$\Rightarrow y = -80$.

$y = -80$ is rejected because the no. cannot be negative.

$\therefore y = 144$.

$\Rightarrow x^2 = 144 = (12)^2$.

$\Rightarrow x = \pm 12$.

i.e., smaller no. is +12, the greater no. is $\frac{1}{8} \times 144 = 18$.

when smaller no. is -12, the greater no. is $\frac{1}{8} \times 144 = 18$.

$\therefore$ the no. are either 12 & 18.

Q8. Let total dist. travelled = 360 km.

Le the uniform speed be x km/h.

Increased speed = (x+5) km/h.

$$\frac{360}{x} - \frac{360}{x+5} = 1.$$

$$\rightarrow \frac{360(x+5) - 360x}{x(x+5)} = 1.$$

$$\rightarrow 360x + 1800 - 360x = x(x+5)$$

$$\rightarrow 1800 = x^2 + 5x \qquad \rightarrow x^2 + 5x - 1800 = 0.$$

$$\rightarrow x^2 + 45x - 40x - 1800 = 0. \qquad \rightarrow x(x+45) - 40(x+45) = 0.$$

$$\rightarrow (x-40)(x+45) = 0.$$

$$x - 40 = 0.$$

$\therefore$ Speed of train = 40 km/h.

Q9. Let the speed of passenger train = x km/h.

then, the Speed of express train be (x+11) km/h.

$$\frac{132}{x} - \frac{132}{x+11} = 1. \rightarrow \frac{132(x+11) - 132x}{x(x+11)} = 1.$$

$$\rightarrow 132x + 1452 - 132x = x^2 + 11x.$$

$$\rightarrow x^2 + 11x - 1452 = 0. \qquad \rightarrow x^2 + 44x - 33x - 1452 = 0$$

$$\rightarrow x(x+44) - 33(x+44) = 0. \qquad \rightarrow (x-33)(x+44) = 0.$$

$$\rightarrow x - 33 = 0.$$

$$\rightarrow x = 33.$$

$\therefore$ Speed of passenger train = 33 km/h.

Speed of express train = 33 + 11 = 44 km/h.

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