Exercise 4.3

1.(i) $2x^2 - 7x + 3$

$\Rightarrow \quad \frac{2x^2}{2} - \frac{7x}{2} + \frac{3}{2}$

$\Rightarrow \quad x^2 - \frac{7x}{2} + \frac{3}{2}$

$\Rightarrow \quad (x)^2 - 2 \cdot x \cdot \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2 = \frac{-3}{2} + \frac{49}{16}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2 = \frac{-24+49}{16}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow x-\frac{7}{4} = \pm\frac{5}{4}$

$\Rightarrow x = \frac{5}{4}+\frac{7}{4} = \frac{12}{4} = 3$

$\Rightarrow x = \frac{-5}{4}+\frac{7}{4} = \frac{2}{4} = \frac{1}{2}$

(ii) $2x^2+x-4 = 0$

$\Rightarrow \frac{2x^2}{2} + \frac{x}{2} - \frac{4}{2} = 0$

$\Rightarrow x^2 + \frac{x}{2} - 2$

$\Rightarrow (x)^2 + 2\cdot x\cdot\frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$

$\Rightarrow \left(x-\frac{1}{4}\right)^2 = 2+\frac{1}{16} = \frac{32+1}{16} = \frac{33}{16}$

$\Rightarrow x-\frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow x = \frac{1}{4}+\frac{\sqrt{33}}{4} = \frac{1+\sqrt{33}}{4}$

$\Rightarrow x = \frac{1}{4}-\frac{\sqrt{33}}{4} = \frac{1-\sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \cdot 2x \cdot \sqrt{3} + (\sqrt{3})^2 = (\sqrt{3})^2 + 3$

$\Rightarrow (2x + \sqrt{3})^2 = -3 + 3$

$\Rightarrow 2x + \sqrt{3} = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$ (does not exist)

Q (i) $2x^2 - 7x + 3 = 0$

$b^2 - 4ac = 49 - 4 \times 3 \times 2$

$= 49 - 24$

$= 25$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{7 \pm \sqrt{25}}{4}$

$x = \frac{7 - 5}{4} = \frac{2}{4} = \frac{1}{2}$

$x = \frac{7 + 5}{4} = \frac{12}{4} = 3$

(ii) $2x^2 + x - 4 = 0$ $\quad b^2 - 4ac = 1 - 4 \times -4 \times 2 = 33$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad x = \frac{-1 + \sqrt{33}}{4}$

$x = \frac{-1 \pm \sqrt{33}}{4}$ $\quad x = \frac{-1 - \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ $\quad b^2 - 4ac = 48 - 4 \times 3 \times 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

$= \frac{-4\sqrt{3} \pm 0}{2}$

(iv) $2x^2 + x + 4 = 0$ $\quad b^2 - 4ac = 1 - 4 \times 4 \times 2 = -31$

does not exist

3. (i) $x - \frac{1}{x} = 3$

$\Rightarrow \frac{x^2 - 1}{x} = 3$

$\Rightarrow x^2 - 3x - 1 = 0$

$b^2 - 4ac = 9 - 4 \times 1 \times -1$

$= 13$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\Rightarrow \frac{x - 7 - x - 4}{x^2 - 7x + 4x - 28} = \frac{11}{30}$

$\Rightarrow \frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$

$\Rightarrow 11x^2 - 33x - 308 = -330$

$\Rightarrow 11x^2 - 33x + 22$

$\Rightarrow x^2 - 3x + 2$

$\Rightarrow x^2 - 1x - 2x + 2$

$\Rightarrow x(x-1) - 2(x-1)$

$x = 1, 2$

4. $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2 + 5x - 3x - 15} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2 + 2x - 15} = \frac{1}{3}$

$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 + 3x - 7x - 21$

$\Rightarrow x(x+3) - 7(x+3)$

$x = 7$

5. $(x+2)(x-3) = 210$

$(x+2)(27-x) = 210$

$27x - x^2 + 54 - 2x = 210$

$-x^2 + 25x + 54 - 210 = 0$

$x^2 - 25x - 54 + 210 = 0$

$x^2 - 25x + 156 = 0$

$x^2 - 13x - 12x + 156$

$x(x-13) - 12(x-13)$

$x = 12, 18$ OR $13 \mid 17$

6. $x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$x^2 - 60x - 2700 = 0$

$x^2 + 30x - 90x - 2700$

$x(x+30) - 90(x+30)$

$x = 90, 120$

7. $A^2 - a^2 = 180$

$a^2 = 8A$

$A^2 - 8A - 180 = 0$

$A^2 + 10A - 18A - 180$

$A(A+10) - 18(A+10)$

$A = 18, 12$

8. $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow \frac{360x + 1800 - 360x}{x^2+5} = 1$

$\Rightarrow x^2 + 5 - 1800 = 0$

$\Rightarrow x^2 - 40x + 45x - 1800$

$\Rightarrow x(x-40) + 45(x-40)$

$\Rightarrow x = 40, 45$

9. $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\frac{x-10+x}{x^2-10x} = \frac{8}{75}$

$\frac{2x-10}{x^2-10x} = \frac{8}{75}$

$150x - 750 - 8x^2 + 80x = 0$

$-8x^2 + 230x - 750$

$8x^2 - 230x + 750$

~~$4x^2 - 80x + 375$~~

$4x^2 - 115x + 375$

$4x^2 - 100x - 15x + 375$

$4x(x-25) - 15(x-25)$

$x = 25, 15$

10. $\frac{132}{x} + \frac{132}{x+11} = 1$

$\frac{132x + 1452 - 132x}{x^2+11x} = 1$

$x^2+11x-1452 = 0$

$x^2 - 33x + 44x - 1452$

$x(x-33) + 44(x-33)$

$x = 33, 44$

11. $A^2 + a^2 = 468$

$4A - 4a = 24$

$A - a = 6$

$A = 6 + a$

$(6+a)^2 + a^2 = 468$

$36 + a^2 + 12a + a^2 = 468$

$2a^2 + 12a - 432 = 0$

$a^2 + 6a - 216 = 0$

$a^2 - 12a + 18a - 216$

$a(a-12) + 18(a-12)$

$a = 12, 18$