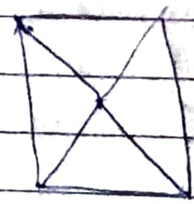
Q4) Given: ABCD is a square.
Diagonals intersect at O.

To prove: we need to prove 3 things
i) The diagonals of a square are equal i.e. AC = BD
ii) bisect each other, i.e. OA = OC & OB = OD
iii) At right angles, any of ∠AOB, ∠BOC, ∠COD, ∠AOD is 90°

Proof:
In △ABC & △DCB
AB = DC (sides of sq. are equal)
∠ABC = ∠DCB (90°)
BC = BC (common)
∴ △ABC ≅ △DCB (By SAS)

AC = DB
Hence the diagonals of a square are equal in length.



Now, AD ∥ BC & AC is a transversal.
(∵ A square is a parallelogram)

∴ ∠1 = ∠3

(Alternate interior angles are equal)

Similarly, $\angle 2 = \angle 4$

Now, in $\triangle OAD$ & $\triangle OCB$, we have $AD = CD$ (sides of square ABCD)

$\angle 1 = \angle 3$ (proved)
$\angle 2 = \angle 4$ (proved)

$\therefore \triangle OAD \cong \triangle OCB$ [By ASA]

$\Rightarrow OA = OC$ & $OD = OB$ (By CPCT)

i.e., the diagonals AC & BD bisect each other at O.

(ii) In $\triangle OBA$ & $\triangle ODA$, we have
$OB = OD$ (proved)
$BA = DA$ (sides of square ABCD]

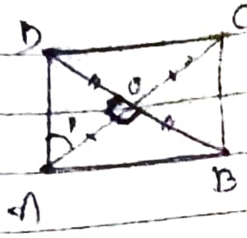
$OA = OA$ (common)
$\therefore \triangle OBA \cong \triangle ODA$ (By SSS)

$\Rightarrow \angle AOB = \angle AOD$ (By CPCT).

$\because \angle AOB$ & $\angle AOD$ form a linear pair.
$\therefore \angle AOB + \angle AOD = 180^\circ$
$\therefore \angle AOB = \angle AOD = 90^\circ$

$\Rightarrow AC \perp BD$

Q5) Let ABCD be a quadrilateral such that diagonals AC & BD are equal & bisect each other at right angles



Now, in ΔAOD & ΔAOB, we have ~~$\angle AOB$~~

$\angle AOD = \angle AOB$ (Each 90°)
AO = AO (common)
OD = OB ($\because$ O is the midpoint of BD)

$\therefore \Delta AOD \cong \Delta AOB$ (By SAS)

$\Rightarrow$ AD = AB (By CPCT) (1)

Similarly we have;

AB = BC (2)
BC = CD (3)
CD = DA (4)

~~also has~~ From (1), (2), (3), (4) we have
AB = BC = CD = DA

$\therefore$ Quadrilateral ABCD have all sides equal.
In ΔAOD & ΔCOB ~~Bay~~, we have
AO = CO (Given)
OD = OB (Given
$\angle AOD = \angle COB$ (Vertically opposite angles)
So, $\Delta AOD \cong \Delta COB$ (By SAS)

$\therefore \angle 1 = \angle 2$ (By CPCT)

But, they form a pair of alternate interior angles.

$\therefore$ AD $\parallel$ BC

Similarly, AB $\parallel$ DC

$\therefore$ ABCD is a parallelogram.

$\therefore$ Parallelogram having all its sides equal is a rhombus.

$\therefore$ ABCD is a rhombus.

Now, in ΔABC & ΔBAD, we have

AC = BD (Given)

BC = AD (proved)

AB = BA (common)

$\therefore \Delta ABC \cong \Delta BAD$ (By SSS)

$\therefore \angle ABC = \angle BAD$ (By CPCT) (5)

Since, AD $\parallel$ BC & AB is a transversal.

$\therefore \angle ABC + \angle BAD = 180^\circ$ (6) (Co-int. angles)

$\Rightarrow \angle ABC = \angle BAD = 90^\circ$ (By (5) & (6))

So, rhombus ABCD is having one angle equal to 90°.

Thus, ABCD is a square.