Key

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Ch-4

Ex-4.3

Q1. find the roots of the following quadratic equation if they exist by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$

A $\frac{2x^2 - 7x + 3}{2} = 0 \Rightarrow \frac{2x^2}{2} - \frac{7x}{2} + \frac{3}{2} = \frac{0}{2}$

$\Rightarrow x^2 - \frac{7x}{2} + \frac{3}{2} = 0$

$\Rightarrow -2ab = -\frac{7x}{2} \quad (a = x)$

$\Rightarrow b = \frac{-7}{4}$

$\Rightarrow x^2 - \frac{7x}{2} + \frac{3}{2} = 0$

$\Rightarrow x^2 + \frac{7x}{2} + \frac{3}{2} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = 0$

$\Rightarrow x^2 + \left(\frac{7}{4}\right)^2 - \frac{7x}{2} + \frac{3}{2} - \left(\frac{7}{4}\right)^2 = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \left(\frac{7}{4}\right)^2 = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0 \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \left(\frac{5}{4}\right)^2$

$x - \frac{7}{4} = \pm \frac{5}{4}$

$x - \frac{7}{4} = \frac{5}{4}$

$\Rightarrow x = \frac{5}{4} + \frac{7}{4}$

$\Rightarrow x = \frac{12}{4} = 3$

$x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = -\frac{5}{4} + \frac{7}{4}$

$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$

(ii) $2x^2+x-4=0$

A. $\frac{2x^2+x-4}{2} = \frac{0}{2}$ $\Rightarrow \frac{2x^2}{2}+\frac{x}{2}-\frac{4}{2}=0$

$\Rightarrow x^2+\frac{x}{2}-2=0$

$2ab = \frac{x}{2}$ $(a=x)$

$2xb = \frac{x}{2}$

$2b = \frac{1}{2}$ $\Rightarrow b = \frac{1}{4}$

$x^2+\frac{x}{2}-2=0$

$x^2+\frac{x}{2}-2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-2-\frac{1}{16}=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2-2-\frac{1}{16}=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = 2+\frac{1}{16}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = \frac{33}{16}$ $\Rightarrow \left(x+\frac{1}{4}\right)^2 = \left(\frac{\sqrt{33}}{4}\right)^2$

$x+\frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$x+\frac{1}{4} = \frac{\sqrt{33}}{4}$

$x = \frac{\sqrt{33}}{4}-\frac{1}{4}$

$x = \frac{\sqrt{33}-1}{4}$

$x-\frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow x = -\frac{\sqrt{33}}{4}-\frac{1}{4}$

$\Rightarrow x = -\frac{\sqrt{33}+1}{4}$

(iii) $4x^2+4\sqrt{3}+3=0$

A. $\frac{4x^2+4\sqrt{3}x+3}{4} = \frac{0}{4}$

$\Rightarrow \frac{4x^2}{4}+\frac{4\sqrt{3}x}{4}+\frac{3}{4}=0$

$\Rightarrow x^2+\sqrt{3}x+\frac{3}{4}=0$

$2ab = \sqrt{3}x$

$2xb = \sqrt{3}x \quad (a = x)$

$\Rightarrow b = \frac{\sqrt{3}}{2}$

$x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$x^2 + \sqrt{3}x + \frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow \left(x^2 + \frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} - \frac{3}{4} = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$

$\left(x + \frac{\sqrt{3}}{2}\right)^2 = \pm 0$

$x + \frac{\sqrt{3}}{2} = +0$	$x + \frac{\sqrt{3}}{2} = -0$
$x + \frac{\sqrt{3}}{2} = 0$	$\Rightarrow x + \frac{\sqrt{3}}{2} = 0$
$x = -\frac{\sqrt{3}}{2}$	$\Rightarrow x = -\frac{\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\frac{2x^2 + x + 4}{2} = \frac{0}{2} \Rightarrow \frac{2x^2}{2} + \frac{x}{2} + \frac{4}{2} = 0$

$\Rightarrow x^2 + \frac{x}{2} + 2 = 0$

$2ab = \frac{1}{2}x$

$2xb = \frac{x}{2} \quad (a = x)$

$2b = \frac{1}{2}$

$b = \frac{1}{4}$

$x^2 + x^3 + 2 = 0$

$x^2 + \frac{x^2}{2} + 3 + \left(\frac{1}{4}\right)^2 + 2 - \left(\frac{1}{4}\right)^2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + 2 - \left(\frac{1}{4}\right)^2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + 2 - \frac{1}{16} = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{-32}{16}$

3. find the roots of the following equations

(i) $x - \frac{1}{x} = 3, \quad x \neq 0$

$x - \frac{1}{x} = 3$

$\Rightarrow \frac{x^2 - 1}{x} = 3 \Rightarrow x^2 - 1 = 3x \Rightarrow x^2 - 3x - 1 = 0$

$a = 1, \ b = -3, \ c = -1$

$D = b^2 - 4ac$

$D = (-3)^2 - 4 \times 1 \times -1$

$D = 13$

$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{13}}{2 \times 1} \Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$

$x = \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \ x \neq -4, 7$

$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30} \Rightarrow \frac{x - 7 - x - 4}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{-11 \times 30}{11} = (x+4)(x-7)$

$\Rightarrow -30 = (x+4)(x-7) \Rightarrow (x+4)(x-7) = -30$

$\Rightarrow x(x-7) + 4(x-7) = -30 \Rightarrow x^2 - 7x + 4x - 28 = -30$

$\Rightarrow x^2 - 3x - 28 = -30 \Rightarrow x^2 - 3x - 28 + 30 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$a = 1, \ b = -3, \ c = 2$

$D = b^2 - 4ac$

$D = (-3)^2 - 4 \times 1 \times 2 \qquad D = 9 - 8 \qquad D = 1$

$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2 \times 1} \Rightarrow x = \frac{3 \pm 1}{2}$

$x = \frac{3+1}{2} \Rightarrow x = \frac{4}{2} = 2$ | $x = \frac{3-1}{2} \Rightarrow x = \frac{2}{2} = 1$

Ans:

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2. Find roots of quadratic equation give Q1. above by applying quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

$a = 2, \quad b = -7, \quad c = 3$

$D = b^2 - 4ac$

$D = (-7)^2 - 4 \times 2 \times 3$

$D = 25$

$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-(-7) \pm \sqrt{25}}{2 \times 2}$

$\Rightarrow x = \frac{7 \pm \sqrt{5^2}}{4} \Rightarrow x = \frac{7 \pm 5}{4}$

$x = \frac{7+5}{4}$	$x = \frac{7-5}{4}$
$x = \frac{12}{4} = 3$	$x = \frac{2}{4} = \frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

$a = 2, \quad b = 1, \quad c = -4$

$D = b^2 - 4ac$

$D = (1)^2 - 4 \times 2 \times -4$

$D = 1 + 32 = 33$

$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{1 \pm \sqrt{33}}{2 \times 2} \Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$a = 4, \quad b = 4\sqrt{3}, \quad c = 3$

$D = b^2 - 4ac$

$D = (4\sqrt{3})^2 - 4 \times 4 \times 3 \Rightarrow D = 4\sqrt{3} \times 4\sqrt{3} - 4 \times 4 \times 3$

$\Rightarrow D = 16 \times 3 - 4 \times 4 \times 3 \Rightarrow D = 48 - 48 = 0$

$x = \frac{-b \pm \sqrt{0}}{2a} \Rightarrow x = \frac{(-4\sqrt{3}) \pm \sqrt{0}}{2 \times 4}$

$\Rightarrow x = \frac{-4\sqrt{3}}{2 \times 4} \Rightarrow x = \frac{-\sqrt{3}}{2}$

4. The sum of the reciprocal of Rehman's age, 3 years ago & 5 years from now is $\frac{1}{3}$. Find his present age.

A Let current age of Rehman = x

3 yrs ago Rehman's age = x − 3

5 yrs from now " " = x + 5

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow (2x+2)3 = 1(x-3)(x+5) \Rightarrow 6x+6 = x(x+5) - 3(x+5)$$

$$\Rightarrow 6x+6 = x^2+5x-3x-15 \Rightarrow 0 = x^2+5x-3x-15-6x-6$$

$$\Rightarrow 0 = x^2+5x-3x-6x+5 \Rightarrow 0 = x^2-4x-21$$

$$\Rightarrow x^2-4x-21 = 0$$

$$x^2+3x-7x-21 = 0$$

$$\Rightarrow x(x+3) - 7(x+3) = 0$$

$$\Rightarrow (x-7)(x+3)$$

$x-7=0$	$x+3=0$
$x=7$	$x=-3$

∴ Rehman's current age = 7

5. In a class test, the sum of Shefali's Mark in Math & English is 30. Had she got 2 marks more in Maths & 3 mark less in English, the product of their marks would have been 210. find her marks in the two subjects

A Let Shefali's mark in Math = x

x + Marks in Eng = 30

⇒ Marks in Eng = 30 − x

$$(x+2)(30-x-3) = 210$$

$$\Rightarrow (x+2)(27-x) = 210 \Rightarrow x(27-x) + 2(27-x) = 210$$

$$\Rightarrow 27x - x^2 + 54 - 2x = 210 \Rightarrow 27x - x^2 + 54 - 2x - 210 = 0$$

$$\Rightarrow -x^2 - 25x + 27x + 54 - 210 = 0 \Rightarrow -x^2 + 25x - 156 = 0$$

$$\Rightarrow 0 = x^2 - 25x + 156$$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 12x - 13x + 156 = 0$

$\Rightarrow x(x-12) - 13(x-12) = 0$

$\Rightarrow (x-13)(x-12) = 0$

$x - 13 = 0$	$x - 12 = 0$
$\Rightarrow x = 13$	$x = 12$

Mark in Math $= x = 12$

" " Eng $= 30 - x$

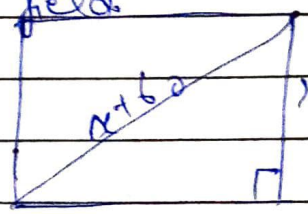
$\Rightarrow 30 - 12 = 18$

Marks in Maths $= x = 13$

" " English $= 30 - x$

$= 30 - 13 = 17$

6. The diagonal of a rectangular field is 60m more than shorter side. If the longer side is 30m more than shorter side, find the sides of the field.



Let shorter side be x

Diagonal $= x + 60$

longer side $= x + 30$

$(x+30)^2 + x^2 = (x+60)^2$

$\Rightarrow x^2 + 60x + 900 + x^2 = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

$D = 3600 - 4(1)(-2700) = 3600 + 10800 = 14400$

$\sqrt{D} = 120$

$x = \frac{60 \pm 120}{2} \Rightarrow x = 90, -30$

Shorter side $= 90$ m, longer side $= 120$ m

7. The difference of squares of two no. is 180. The square of the smaller no. is 4 times the

large no. find the two no.

A. The difference of squares of two no is 180

$(x > y)$

$x^2 - y^2 = 180$

$y^2 = 8x$

$x^2 - 8x - 180 = 0$

$D = b^2 - 4ac$

$= (-8)^2 - 4 \times 1 \times (-180)$

$= 64 + 720 = 784 > 0$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{8 \pm 28}{2 \times 1} = \frac{36}{2}$ or $\frac{-20}{2} = -10$

$= 18$

$y^2 = 8 \times 18$

$= 4 \times 36$

$= (2 \times 6)^2$

$= y = 12$

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hr less for same journey. Find the speed of the train.

A. Let speed of train be $x\ km\,h^{-1}$

Time $= \frac{\text{Distance}}{\text{Speed}}$ $t_1 = \frac{360}{x}$ $t_2 = \frac{360}{x+5}$

$\frac{360}{x} - \frac{360}{x+5} = 1$ $\Rightarrow 36\left[\frac{x+5-x}{x(x+5)}\right] = 1$

$\Rightarrow 360 \times 5 = x^2 + 5x$ $\Rightarrow x^2 + 5x - 1800 = 0$

$D = b^2 - 4ac$ $= 25 + 7200 = 7225 > 0$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{7225}}{2} = \frac{-5 \pm 85}{2}$

$= \frac{-5+85}{2}$ or $\frac{-5-85}{2} = 40$ or -45

$x = 40$, Speed of train $= 40$

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hr. The tap of larger diameter takes 10 hr less than the smaller one to fill the tank. Find the time taken

which each tap can separately fill the tank

A. Smaller diameter tap?

Let larger diameter tap = $(x-10)$ hr

Time taken by both taps to fill = $9\frac{3}{8} = \frac{75}{8}$

Tank fill by smaller tap in 1 hr = $\frac{1}{x}$

" " " larger " " " = $\frac{1}{x-10}$

$$\text{So } \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75} \Rightarrow \frac{2x-10}{x^2-10x} = \frac{8}{75}$$

$$\Rightarrow 75(2x-10) = 8(x^2-10x)$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x \Rightarrow -8x^2 + 150x + 80x - 750 = 0$$

$$\Rightarrow -8x^2 + 230x - 750 = 0 \Rightarrow 0 = 8x^2 - 230x + 750$$

$$D = b^2 - 4ac$$

$$D = (-230)^2 - 4 \times 8 \times (750)$$

$$D = 52900 - 24000 \quad D = 28900$$

$$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-(-230) \pm \sqrt{28900}}{2 \times 8}$$

$$\Rightarrow x = \frac{230 \pm \sqrt{28900}}{16} \Rightarrow x = \frac{230 \pm \sqrt{289 \times 100}}{16}$$

$$\Rightarrow x = \frac{230 \pm \sqrt{17^2} \times \sqrt{10^2}}{16} \Rightarrow x = \frac{230 \pm 17 \times 10}{16}$$

$$\Rightarrow x = \frac{230 \pm 170}{16}$$

$$x = \frac{230+170}{16} \qquad x = \frac{230-170}{16}$$

$$\Rightarrow x = \frac{400}{16} \, 25 \qquad \Rightarrow x = \frac{60}{16} \, \frac{15}{4}$$

Time taken by small tap = x = 25 hr

" " " large " = $x - 10 \Rightarrow 25 - 10 = 15$ hr

Time taken by small tap = $x = \frac{15}{4}$ hr

" " " large " = $x - 10$

$= \frac{15}{4} - 10 = -\frac{25}{4}$

12. Let passenger train be = x km/hr
" Express " " = x+11 km/hr
Distance = 132 km
$t_1 = \frac{132}{x}$ $t_2 = \frac{132}{x+11}$

$\frac{132}{x} - \frac{132}{x+11} = 1 \Rightarrow 132\left[\frac{1}{x} - \frac{1}{x+1}\right] = 1$

$\Rightarrow 132\left[\frac{x+11-x}{x^2+11x}\right] = 1 \Rightarrow 132 \times 11 = x^2+11x$

$\Rightarrow x^2+11x-1452 = 0$

$D = b^2-4ac = (11)^2 - 4\times1\times(-1452)$
$= 121 + 5808 = 5929$

$x = \frac{-11 \pm \sqrt{5929}}{2\times1} \Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2}$

$\Rightarrow x = \frac{-11 \pm 77}{2}$

$x = \frac{-11-77}{2}$	$x = \frac{-11+77}{2}$
$x = \frac{-88}{2} = -44$	$x = \frac{66}{2} = 33$

$x = 33$ = Speed of passenger train
Express Train = $x+11 = 33+11 = 44$ km/hr

11. Sum of the areas of two squares is 468 m². If the difference of their perimeter is 24 m. Find the sides of the two squares.

A Let the side of first square be a m & second be b m
Area of first square = a^2 sq m
" " second " = b^2 sq m
Perimeter = 4a & 4b
$4a - 2b = 24 \Rightarrow a-b = 6$ — (1)

$a^2 + b^2 = 468$ ——— (2)

$(b+6)^2 + b^2 = 468$

$\Rightarrow b^2 + 12b + 36 + b^2 = 468$

$\Rightarrow 2b^2 + 12b + 36 - 468 = 0$

$\Rightarrow b^2 + 6b - 216 = 0$

$\Rightarrow b^2 + 18b - 12b - 216 = 0$

$\Rightarrow b(b+18) - 12(b+18) = 0$

$\Rightarrow (b+18)(b-12) = 0$

$b + 18 = 0$ | $b - 12 = 0$

$\Rightarrow b = -18$ | $\Rightarrow b = 12$

$b = 12\,m$