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ch-5 Arithmetic Progression

Ex. 5.1

1. In which of following situations, does the list of numbers formed make an arithmetic progression? Why?

(i) The taxi fare after each km when the fare is ₹ 15 for the first km & ₹ 8 for each additional km

A Taxi fare for 1st km = 15
" " for 2nd km = 15 + 8 = 23
" " " 3rd km = 23 + 8 = 31
" " " 4th km = 31 + 8 = 39

So, it is an AP. Every term is 8 more than the preceding term

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time

A let volume of air in a cylinder be V &
pump removes $\frac{1}{4}$ of air remaining in cylinder at a time

$\frac{3}{4}$ part of air will remain

$\frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^2 V \ldots\ldots$

Don't have same difference between them.

∴ It is not an AP

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for

the first metres is rised by ₹50 for each subsequent metre.

A Cost of digging first metre = ₹150
" " " 2 metre = ₹150 + ₹50 = ₹200
" " " 3 metre = ₹200 + ₹50 = ₹250

Series is 150, 200, 250, ----
Difference between second & first term = 200 - 150 = 50
Difference between third & second term = 250 - 200 = 50
The Difference is same. So it is an AP.

(iv) The amount of money in the account every year, when ₹10000 is deposited at compound interest of 4% per annum

A Original amount = ₹ 10000

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

$$10000\left(1+\frac{4}{100}\right)^1 = 10000 \times \frac{104}{100} = 10400$$

$$10000\left(1+\frac{4}{100}\right)^2 = 10000 \times \left(\frac{104}{100}\right)^2 = 11664$$

$$10000\left(1+\frac{4}{100}\right)^3 = 10000\left(\frac{104}{100}\right)^3 = 12597.12$$

The series is 10400, 11664, 12597.12, ----
Difference between second & first term = 11664 - 10400 = 864
Difference between third & second term = 12597.12 - 11664 = 933.12
The difference is not same.
864 ≠ 933.12. So it is not an AP.

2. Write first four term of AP, when the first term a & common difference d are given as follow

(i) $a = 10$, $d = 10$

A $a_1 = a = 10$

$a_2 = 10 + 10 = 20$

$a_3 = 20 + 10 = 30$

$a_4 = 40 + 10 = 50$

So the series will be 10, 20, 30, 40, 50 - - - - - . First four term of AP will be 10, 20, 30, 40

(ii) $a = -2$, $d = 0$

A. $a_1 = a = -2$

$a_2 = -2 + 0 = -2$

$a_3 = -2 + 0 = -2$

$a_4 = -2 + 0 = -2$

The series will be $-2, -2, -2, -2$ ... first four term of this AP will be $-2, -2, -2$ & -2

(iii) $a = 4$, $d = -3$

$a_1 = a = 4$

$a_2 = 4 - 3 = 1$

$a_3 = 1 - 3 = -2$

$a_4 = -2 - 3 = -5$

The series will be $4, 1, -1, -5$, ... So first four term of AP will be $4, 1, -2, -5$

(iv) $a = -1$, $d = \frac{1}{2}$

A $a_1 = a = -1$

$a_2 = -1 + \frac{1}{2} = -\frac{1}{2}$

$a_3 = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 = 0 + \frac{1}{2} = \frac{1}{2}$

So, series will be $-1, -\frac{1}{2}, 0, \frac{1}{2}$, ... first four term is $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(v) $a = -1.25$, $d = 0.25$

A $a_1 = a = -1.25$

$a_2 = -1.25 - 0.25 = -1.50$

$a_3 = -1.50 - 0.25 = -1.75$

$a_4 = -1.75 - 0.25 = -2.00$

So first four term of AP is $-1.25, -1.50, -1.75, -2.00$

3. For the following APs, write the first term & common difference.

(i) $3, 1, -1, -3, \ldots$

A. first term, $a = 3$
Common difference $= 1 - 3 = -2$

(ii) $-5, -1, 3, 7, \ldots$

A. first term $a = -5$
Common difference $= -1 - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$

A. first term, $a = \frac{1}{3}$
Common difference $= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $0.6, 1.7, 2.8, 3.9, \ldots$

A. first term, $a = 0.6$
common difference, $d = 1.7 - 0.6 = 1.1$

4. Which of following are APs? If they form AP, find the common difference & write three more term

(i) $2, 4, 8, 16 \ldots$

A. $a_2 - a_1 = 4 - 2 = 2$
$a_3 - a_2 = 8 - 4 = 4$
$a_4 - a_3 = 16 - 8 = 8$

$a_{k+1} - a_k$ is not same every time. Difference is not same. It is not an AP

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

Difference is same, so it is an AP

Three more term are

$a_5 = \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$

$a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) $-1.2, -3.2, -5.2, -7.2, \ldots\ldots$

A. $a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

Difference is same, so it is an AP

Three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) $-10, -6, -2, 2, \ldots\ldots$

A $a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

Difference is same, so it is an AP

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots\ldots$

A $a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = (3 + 2\sqrt{2}) - (3 + \sqrt{2}) = \sqrt{2}$

$a_4 - a_3 = (3 + 3\sqrt{2}) - (3 + 2\sqrt{2}) = \sqrt{2}$

Difference is same, so it is an AP

Three more terms:-

$a_5 = (3+\sqrt{2}) + \sqrt{2} = 3+4\sqrt{2}$

$a_6 = (3+4\sqrt{2}) + \sqrt{2} = 3+5\sqrt{2}$

$a_7 = (3+5\sqrt{2}) + \sqrt{2} = 3+6\sqrt{2}$

(iii) 0.2, 0.22, 0.222, 0.2222 ——

A $a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.002$

Difference is not same. So it is not an AP

(iv) 0, -4, -8, -12, ——

A $a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

Difference is same, so it is an AP

~~Difference~~

$a_5 = -12 - 4 = -16$

$a_6 = -16 - 4 = -20$

$a_7 = -20 - 4 = -24$

(v) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ ——

A $a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$

Difference is same, it is an AP.

Three more terms are

$a_5 = \left(-\frac{1}{2}\right) - 0 = -\frac{1}{2}$

$a_6 = \left(-\frac{1}{2}\right) - 0 = -\frac{1}{2}$

$a_7 = \left(-\frac{1}{2}\right) - 0 = -\frac{1}{2}$

(vi) 1, 3, 9, 27, ——

A $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

It is not an AP, difference is not same

(x) $a, 2a, 3a, 4a, \cdots$

A $a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Difference is same, so it is an AP

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4, \cdots$

A $a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

Difference is not same. It is not an AP

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \cdots$

A $a_2 - a_1 = \sqrt{8} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

It is an AP

Next three term are $\sqrt{50}, \sqrt{72}, \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \cdots$

A. $a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}\times\sqrt{2} - \sqrt{3} = \sqrt{3}(\sqrt{2}-1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3}\times 3 = \sqrt{3}(2 - \sqrt{3})$

Difference is not same. So it is not an AP

(xiv) $1^2, 3^2, 5^2, 7^2, \ldots$

A) $1, 9, 25, 49, \ldots$

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

Difference is not same. So it is not an AP

(xv) $1^2, 5^2, 7^2, 73, \ldots$

A) $1, 25, 49, 73, \ldots$

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

Difference is same. It is an AP

Three more terms

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$