Ex-5.3

1. Find the sum of the following APs:

(i) 2, 7, 12, ... to 10 terms

A. $a = 2$, $d = 5$, $n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{\cancel{10}^5}{\cancel{2}}[2(2) + (10-1) \times 5] = 5[4 + (9) \times 5] = 5 \times 49$
$= 245$

(ii) -37, -33, -29, ... to 12 terms

A. $a = -37$, $d = 4$, $n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{12} = \frac{12}{2}[2(-37) + (12-1)4]$

$= 6(-74 + 11 \times 4) = 6(-30) = -180$

(iii) 0.6, 1.7, 2.8, ... to 100 terms

A. $a = 0.6$, $d = 1.1$, $n = 100$ terms

$S_{100} = \frac{100}{2}[2(0.6) + (100-1)1.1] = 50[1.2 + (99) \times (1.1)]$

$= 50[110.1] = 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$ to 11 terms

A. $a = \frac{1}{15}$, $d = \frac{1}{60}$, $n = 11$

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$

$= 11 \cdot 2\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$

2. Find the sums of given below

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

A. $a = 7$

$l = 84$

$d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$

$a_n = a + (n-1)d$

$84 = 7 + (n-1) \times \frac{7}{2} \Rightarrow 77 = (n-1) \times \frac{7}{2}$

$\Rightarrow \frac{77 \times 2}{7} = (n-1) \Rightarrow 22 = n-1$

$\Rightarrow n = 22+1 \Rightarrow n = 23$

$S_{13} = \frac{n}{2}(a+l) \Rightarrow S_n = \frac{23}{2}(7+84)$

$= \frac{23 \times 91}{2} = \frac{2093}{2} = 1046\frac{1}{2}$

(ii) $34 + 32 + 30 + \dots + 10$

A $a = 34$, $d = -2$, $l = 10$

$a_n = a + (n-1)d$

$10 = 34 + (n-1) \times -2 \Rightarrow -24 = (n-1) \times 2$

$\Rightarrow \cancel{\frac{-24}{-2}} = (n-1) \Rightarrow 12 = n-1 \Rightarrow n = 13$

$S_{13} = \frac{n}{2}(a+l) \Rightarrow S_{13} = \frac{13}{2}(34+10) \Rightarrow S_{13} = \frac{13}{2}(44)$

$\Rightarrow S_{13} = 286$

(iii) $-5 + (-8) + (-11) + \dots + (-230)$

A. $a = -5$, $d = -3$, $l = -230$

$a_n = a + (n-1)d$

$\Rightarrow -230 + 5 = (n-1) \times 3 \Rightarrow -225 = (n-1) \times -3$

$\Rightarrow \cancel{\frac{-225}{-3}} = (n-1) \Rightarrow 75 = n-1 \Rightarrow n = 76$

$S_{76} = \frac{n}{2}(a+l)$

$S_n = \frac{76}{2}(-5+(-230)) \Rightarrow S_n = 38(-235)$

$\Rightarrow S_n = -8930$

3. In an AP

(i) Given $a=5$, $d=3$, $a_n=50$, find n & S_n

A. $a_n = a+(n-1)d$

$\Rightarrow 50 = 5+(n-1)\times 3 \Rightarrow 50 = 5+3n-3$

$\Rightarrow 50 = 2+3n \Rightarrow 50-2 = 3n \Rightarrow 48 = 3n$

$\Rightarrow \frac{48}{3} = n \quad = n = 16$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{16}{2}[2(5)+(16-1)3] = 8(10+15\times 3) = 8\times 55$

$= 440$

(ii) Given $a=7$, $a_{13}=35$, find d & S_{13}

A. $a_n = a+(n-1)d$

$\Rightarrow 35 = 7+(13-1)d \Rightarrow 35 = 7+12d$

$\Rightarrow \cancel{35 = 7+(13} \; 35-7 = 12d \Rightarrow 28 = 12d \Rightarrow d = \cancel{\frac{28}{12}}$

$\frac{7}{3}$

$S_n = \frac{n}{2}(a+l)$

$= \frac{13}{2}(7+35) = \frac{13}{2}\times 42 = 13\times 21 = 273$

(iii) Given $a_{12} = 37$, $d=3$, find a & S_{12}

A. $a_n = a+(n-1)d$

$\Rightarrow 37 = a+(12-1)3 \Rightarrow 37 = a+11\times 3 \Rightarrow 37 = a+33$

$\Rightarrow 37-33 = a \Rightarrow 4 = a$

$S_n = \frac{n}{2}(a+l)$

$S_{12} = \frac{12}{2}(4+37) \Rightarrow \frac{12}{2}\times 41 = 6\times 41 = 246$

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d & a_{10}.

A. $a_3 = 15$, $S_{10} = 125$

$a_n = a+(n-1)d$

$a_3 = a+(3-1)d$ ⇒ $15 = a+2d$ —— (1)

$S_n = \frac{n}{2}[2a+(n-1)d]$ ⇒ $S_{10} = \frac{10}{2}[2a+(10-1)d]$

$125 = 5(2a+9d)$

$25 = 2a+9d$ —— (2)

$30 = 2a+4d$ —— (3)

$-5 = 5d$ ⇒ $d = -1$

$15 = a+2(-1)$ ⇒ $15 = a-2$ ⇒ $a = 17$

$a_{10} = a+(10-1)d$ ⇒ $a_{10} = 17+(9)(-1) = 8$

(v) Given $d=5$, $S_9 = 75$, find a & a_9

A. $S_n = \frac{n}{2}[2a+(n-1)d]$

⇒ $S_9 = \frac{9}{2}[2a+(9-1)5]$ ⇒ $75 = \frac{9}{2}(2a+40)$

⇒ $25 = 3(a+20)$ ⇒ $25 = 3a+60$ ⇒ $3a = 25-60$

⇒ $a = \frac{-35}{3}$

$a_n = a+(n-1)d$

$a_9 = a+(9-1)(5)$

$= \frac{-35}{3}+8(5) = \frac{-35+120}{3} = \frac{85}{3}$

(vi) Given $a=2$, $d=8$, $S_n = 90$, find n & a_n

A. $S_n = \frac{n}{2}[2a+(n-1)d]$

⇒ $90 = \frac{n}{2}[4+(n-1)8]$ ⇒ $180 = 8n^2-4n$

⇒ $8n^2-4n-180=0$ ⇒ $2n^2-n-45=0$

⇒ $2n^2-10n+9n-45=0$

$\Rightarrow 2n(n-5)-9(n-5)$ $= (2n-9)(n-5)$

$n=5$

$a_n = a+(n-1)d$

$a_n = 2+(5-1)8$ $\Rightarrow a_n = 2+32 \Rightarrow a_n = 34$

(vii) Given $a=8$, $a_n = 62$, $S_n = 210$, find n & d

$l = 62$

A $S_n = \frac{n}{2}(a+l)$

$\Rightarrow 210 = \frac{n}{2}(8+62)$ $\Rightarrow 210\times 2 = n\times 70$

$\Rightarrow 420 = n\times 70$ $\Rightarrow \frac{420}{70} = n$ $\Rightarrow n=6$

$a_n = a+(n-1)d$

$\Rightarrow 62 = 8+(6-1)d$ $\Rightarrow 62 = 8+5d$ $\Rightarrow 62-8 = 5d$

$\Rightarrow 54 = 5d$ $\Rightarrow d = \frac{54}{5}$

(viii) Given $a_n = 4$, $d=2$, $S_n = -14$, find n & a

A $a_n = a+(n-1)d$

$4 = a+(n-1)2$ $\Rightarrow 4 = a+2n-2$ $\Rightarrow a+2n = 6$

$a = 6-2n$ —— (1)

$S_n = \frac{n}{2}[a+a_n]$ $\Rightarrow -14 = \frac{n}{2}[a+4] \Rightarrow -28 = n(a+4)$

$\Rightarrow -28 = n(6-2n+4)$ $\Rightarrow -28 = n(-2n+10)$

$\Rightarrow -28 = -2n^2+10n$ $\Rightarrow 2n^2-10n-28=0 \Rightarrow n^2-5n-14=0$

$\Rightarrow n^2-7n+2n-14=0$

$\Rightarrow n(n-7)+2(n-7)=0$

$\Rightarrow (n-7)(n+2)=0$

$n=7$, $n=-2$ (n can neither be negative)

$n=7$

$a = 6-2n$

$a = 6-2(7)$

$= 6-14$

$= -8$

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d

A $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 192 = \frac{8}{2}[2 \times 3 + (8-1)d]$ $\Rightarrow 192 = 4[6 + 7d]$

$\Rightarrow 48 = 6 + 7d$ $\Rightarrow 42 = 7d$ $\Rightarrow \therefore d = 6$

(x) Given $l = 28$, $S = 144$ & there are total 9 term, find a

A $S_n = \frac{n}{2}(a + l)$ $\Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow \frac{144 \times 2}{9} = a + 28$ $\Rightarrow 32 = a + 28$ $\Rightarrow 32 - 28 = a$

$\Rightarrow a = 4$