Hw
7.7.21

4. How many terms of AP: 9, 17, 25 ... must be taken to give a sum of 636?

A. $a = 9$, $d = 8$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 636 = \frac{n}{2}[2 \times a + (n-1)]$

$\Rightarrow 636 = \frac{n}{2}[18 + (n-1)8]$ $\Rightarrow 636 = n[9 + 4n - 4]$

$\Rightarrow 636 = n(4n+5)$ $\Rightarrow 636 = 4n^2 + 5n$

$\Rightarrow 4n^2 + 5n - 636 = 0$

$= 4n^2 + 53n - 48n - 636$

$= n(4n+53) - 12(4n+53)$

$= (4n+53)(n-12)$

$= n = \frac{-53}{4}$ or $n = 12$

5. The first term of an AP is 5, the last term is 45 & sum is 400. Find the no. of terms

& the common difference.

A $a = 5, \quad l = 45, \quad S_n = 400$

$S_n = \frac{n}{2}(a+l)$

$\Rightarrow 400 = \frac{n}{2}(5+45) \Rightarrow 400 = \frac{n}{2} \times 50 \Rightarrow \frac{400 \times 2}{50} = n$

$\Rightarrow 8 \times 2 = n \Rightarrow 16 = n$

$d = ? \quad S_n = \frac{n}{2}\{2a + (n-1)d$

$\Rightarrow 400 = \frac{16}{2}[2 \times 5 + (16-1)d]$

$\Rightarrow 400 = 8 \times (10 + 15d) \Rightarrow \frac{400}{8} = 10 + 15d$

$\Rightarrow 50 = 10 + 15d \Rightarrow 50 - 10 = 15d \Rightarrow 40 = 15d$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

6. The first & last term of the AP are 17 & 350. If the common difference is 9, how many terms are there & what is their sum?

A $a = 17, \quad l = 350, \quad d = 9$

$a_n = a + (n-1)d$

$350 = 17 + (n-1) \times 9 \Rightarrow 350 - 17 = (n-1)9$

$\Rightarrow 333 = (n-1)9 \Rightarrow \frac{333}{9} = n-1 \Rightarrow 37 = n-1 \Rightarrow n = 38$

$\text{Sum} = \frac{38}{2}(17 + 350) = 19 \times 367 = 6973$

7. Find the sum of first 22 terms of a AP in $d = 7$ & 22^{nd} term is 149

A $a_{22} = 149 \quad d = 7$

$a_n = a + (n-1)d$

$\Rightarrow 149 = a + (22-1)7 \Rightarrow 149 = a + 21 \times 7$

$\Rightarrow 149 = a + 147 \Rightarrow 149 - 147 = a \Rightarrow a = 2$

$S_n = \frac{n}{2}(a+l) \Rightarrow \frac{22}{2}(2 + 149) = 11 \times 151 = 1661$

8. Find the sum of first 51 terms of an AP whose second & third terms are 14 & 18.

A. $n = 51$, $a_2 = 14$, $a_3 = 18$

$a_2 = a + (2-1)d$
$\Rightarrow 14 = a + d$
$\Rightarrow a = 14 - d$ — (1)

$a_3 = a + (3-1)d$
$\Rightarrow 18 = a + 2d$
$\Rightarrow a = 18 - 2d$ — (2)

$14 - d = 18 - 2d$
$2d - d = 18 - 14 \Rightarrow d = 4$
$a = 14 - d \Rightarrow a = 14 - 4 \Rightarrow a = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$
$n = 51$, $a = 10$, $d = 4$
$S_{51} = \frac{51}{2}(2 \times 10 + (51-1) \times 4 \Rightarrow S_{51} = \frac{51}{2} \times 220$
$\Rightarrow S_{51} = 51 \times 110 = 610$

Q. If the sum of first 7 terms of an AP is 49 & that of 17 terms is 289, find the sum of first n terms

A. $S_7 = 49$, $S_{17} = 289$, $S_n = ?$

$S = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow \frac{7}{2}(2a + 6d) = 49$
$\Rightarrow \frac{7}{2} \times 2(a + 3d) = 49$
$\Rightarrow a + 3d = 7$

$\frac{17}{2}(2a + 16d) = 289$
$\Rightarrow \frac{17}{2} \times 2(a + 8d) = 289$
$\Rightarrow a + 8d = \frac{289}{17} = 17$
$\Rightarrow a + 8d = 17$

$a + 8d = 17$
$-\ a + 3d = 7$
$5d = 10 \Rightarrow d = 2$

$a + 3d = 7$
$\Rightarrow a + 3(2) = 7 \Rightarrow a + 6 = 7 \Rightarrow a = 1$
$S_n = \frac{n}{2}[2 \times 1 + 2(n-1)] \Rightarrow \frac{n}{2}(\cancel{2} + 2n - \cancel{2})$
$= \frac{n \times 2n}{2} = n^2$

10. Show that $a_1, a_2 \ldots a_n$, form an AP where a_n is defined as below. Also find 15 term

(i) $a_n = 3+4n$

$a_1 = 3+4(1) = 7$

$a_2 = 3+4(2) = 11$

$a_3 = 3+4(3) = 15$

$d = a_3 - a_2 = 15-11 = 4$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{15} = \frac{15}{2}[2\times7+(15-1)4] = \frac{15}{2}(14+56)$

$= \frac{15}{2} \times 70 = 525$

(ii) $a_n = 9-5n$

$a_1 = 9-5(1) = 4$

$a_2 = 9-5(2) = -1$

$a_3 = 9-5(3) = -6$

$d = -6-(-1) = -5$

$a = 4, \quad n = 15$

$S_{15} = \frac{15}{2}[2\times4+(15-1)-5]$

$= \frac{15}{2}(8-70) = \frac{15}{2} \times (-62) = -465$

11. If the sum of first n term of AP is $4n-n^2$, what is the first term. What is the sum of first two term? What is the second term? find 3rd, 10th & n^{th} term

A. $S_n = 4n-n^2$, $n=1$

$S_1 = 4\times1-(1)^2$

$= 4-1 = 3$

| $S_2 = 4\times2-(2)^2$

$= 8-4 = 4$

Sum of first two term $= 4$

$S_2 = a_1 + a_2$

$S_2 = 3 + a_2$

$4 = 3 + a_2$ $\Rightarrow a_2 = 4-3 = 1$

$d = 1 - 3 = -2$

AP = 3, 1, -1, -3, ...

3rd term	10th term	nth term
$a_3 = a + (3-1)d$	$a_{10} = a + 9d$	$a_n = 3 + (n-1)(-2)$
$= a + 2d$	$= 3 + (9) \times 2$	$= 3 - 2n + 2$
$= 3 \times 2(-2)$	$= 3 - 18 = 15$	$= 5 - 2n$
$= 3 - 4 = 1$		

12. Find the sum of first 40 positive integers divisible by 6

A. 6, 12, 18, 24, ...

$S_n = \frac{n}{2} [2a + (n-1)d$

$n = 40, \quad a = 6, \quad d = 6$

$S_n = \frac{40}{2} (2 \times 6 + (40-1) \times 6$

$S_n = 20 (12 + 39 \times 6) \Rightarrow S_{12} = 20 (12 + 234)$

$S_n = 4920$

13. Find the sum of first 15 multiple of 8

Multiple of 8 are 8, 16, 24, ...

A. $n = 15, \quad a = 8, \quad d = 8$

$S_n = \frac{n}{2} [2a + (n-1)d]$

$\Rightarrow \frac{15}{2} = (2 \times 8 + (15-1) \times 8) \Rightarrow \frac{15}{2} (16 + 14 \times 8)$

$\Rightarrow \frac{15}{2} (16 + 112) = \frac{15}{2} \times 128$

$= 960$

14. Find the sum of the odd no. between 0 & 50

A. odd no. between 0 to 50 are

1, 3, 5, 7, 9, ... 49

$a = 1, \quad d = 2, \quad a_n = 49$

$a_n = a + (n-1)d$

$\Rightarrow 49 = 1 + (n-1)2 \qquad \Rightarrow 48 = (n-1)2 \Rightarrow n = 25$

$S_{25} = \frac{25}{2}(1+49) = 625$

15. A Penalty for 1^{st} day = 200
" " 2^{nd} day = 250
" " 3^{rd} day = 300

200, 250, 300, ...
d = 50

$S_n = \frac{n}{2}[2a+(n-1)d]$
n = 30, a = 200

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{30}{2}[2\times 200 + (30+1)\times 50]$

$= 15(400+1450) = 15\times 1850 = 27750$

16. let first prize be a
" second prize be a-20
" Third prize be (a-20)-20 = a-40

a, a-20, a-40

a = a d = (a-20) - a = a-20-a = -20

$S_n = 700$

$700 = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 700 = \frac{7}{2}(2a+6\times -20)$

$\Rightarrow 700 = \frac{7}{2}(2a-120)$

$\Rightarrow 700 \times \frac{2}{7} = 2a-120$

$\Rightarrow 200 = 2a-120$ $\Rightarrow 2a = 200+120$

$\Rightarrow 2a = 320$ $\Rightarrow a = \cancel{\frac{320}{2}}$ 160

1^{st} prize = a = ₹160
2^{nd} " = a-20 = ₹140
3^{rd} " = a-40 = ₹120

17A. Trees planted in class I = 1×3 = 3
" " " " II = 2×3 = 6

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Trees planted in class III = 3 × 3 = 9
" " " " XII = 12 × 3 = 36

3, 6, 9, ..., 36
n = 12, d = 3
$S_n = \frac{n}{2}(a + l)$
$= \frac{\cancel{12}^{6}}{\cancel{2}}(3 + 36) = 6 \times 39 = 234$

18A. Circumference of circle = $2\pi r$
" of semicircle = $\frac{1}{2} \times 2\pi r = \pi r$
" " 1st semicircle = $\pi \times 0.5$
" " 2nd " = $\pi \times 1.0$
" " 3rd " = $\pi \times 1.5$
" " 13th " = $\pi \times 6.5$

$\pi \times 0.5, \pi \times 1.0, \pi \times 1.5, ..., \pi \times 6.5$
= 0.5, 1.0, 1.5, ..., 6.5
$S_n = \frac{n}{2}(a + l)$
$= \frac{13}{2}(0.5 + 6.5)$
$= \frac{13}{2} \times 7 = \frac{91}{2} = 45.5$

length of spiral = $\pi \times S$
$= \frac{22}{7} \times 45.5$
$= 22 \times 6.5 = 143$

19A. No. of logs in 1st row = 20
" " " " 2nd " = 19
" " " " 3rd " = 18

a = 20, d = -1
$S_n = 200$
$S_n = \frac{n}{2}(2a + (n-1)d)$
$200 = \frac{n}{2} \times (2 \times 20 + (n-1) \times -1)$
$\Rightarrow 200 = \frac{n}{2}(40 - n + 1) \Rightarrow 200 = \frac{n}{2} \times (41 - n)$

$\Rightarrow 200 \times 2 = n(41-n)$

$n^2 - 41 + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n-16) - 25(n-16) = 0$

$(n-25)(n-16) = 0$

$n = 25$ or $n = 16$

$a_n = a + (n-1)d$

$a_{16} = 20 + (16-1)(-1)$

$a_{16} = 20 \times 15 \times -1$

$a_{16} = 20 - 15$

$= 5$

$a_{25} = 20 + (25-1) - 1$

$a_{25} = 20 + 24 \times (-1)$

$a_{25} = 20 - 24$

$a_{25} = -4$

No. of rows $= 16$

" " logs in top row $= a_n = 5$

20 A. $a = 5$

$d = 1.75$

$a_n = 20.75$

$a_n = a + (n-1)d$

$20.75 = 5 (n-1) 1.75$

$\Rightarrow 15.75 = (n-1) 1.75$

$(n-1) = \frac{15.75}{1.75} = \cancel{\frac{1575}{175}} = \cancel{\frac{63}{7}}$ 9

$n - 1 = 9$

$n = 10$