HW 03/07/21

EXERCISE - 5.3

① (i) 2, 7, 12, ... to 10 terms.

Here,

$a = 2,\ d = a_2 - a_1$, $n = 10$

$7 - 2 = 5$

We know that,

$\Rightarrow S_{10} = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2(2) + (10-1)5]$

$\Rightarrow S_{10} = 5[4 + 45]$

$\Rightarrow S = 5 \times 49$

$\Rightarrow S_{10} = 245$

(ii) -37, -33, -29 .... to 12 terms.

In the reversing order we can write these terms as - 7, 3, -1, -5 ...

Here,

$a = 7,\ d = a_2 - a_1$, $n = 12$.

$3 - 7 = (-4)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{12}{2}[2 \times 7 + (12-1) - 4]$

$S_{12} = 6[14 + 11 \times (-4)]$

$\therefore S_{12} = 6 \times 30 = 180$.

(iii) $0.6, 1.7, 2.8 \ldots$ to 100 terms.

Here,

$a = 0.6, \; d = 1.7 - 0.6, \; n = 100.$
$= 1.1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2 \times 0.6 + (100-1) 1.1]$

$S_{100} = 50[1.2 + 108.9]$

$S_{100} = 50 \times 110.1 \times 50$

$S_{100} = 5,505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ to 11 terms.

Here,

$a = \frac{1}{15}, \; d = \frac{1}{12} - \frac{1}{15}, \; n = 11$
$= \frac{1}{60}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{11} = \frac{11}{2}\left[2 \times \frac{1}{15} + (11-1)\frac{1}{60}\right]$

$\Rightarrow S_{11} = \left[\frac{11}{2} \times \left(\frac{2}{15} + 10 \times \frac{1}{60}\right)\right]$

$\Rightarrow S_{11} = \left[\frac{11}{2} \times \left(\frac{2}{15} + \frac{1}{6}\right)\right]$

$\Rightarrow S_{11} = \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}$

(2) (i) $7 + \frac{21}{2} + 14 + \cdots + 84$.

Here, $a = 7$, $d = a_2 - a_1$, $n = ?$

$\frac{21}{2} - \frac{14}{2} = \frac{21-14}{2} = \frac{7}{2}$

So, by applying,

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$84 - 7 = (n-1)\frac{7}{2}$

$\cancel{77}^{11} \times \frac{2}{\cancel{7}} = (n-1)$

$n = 22 + 1$

$n = 23$

We know that,

$S_n = \frac{n}{2}(a + l)$

$S_{23} = \frac{23}{2}(7 + 84)$

$S_{23} = \frac{23}{2}(91)$

$S_{23} = \frac{2093}{2}$

(ii) $34 + 32 + 30 + \cdots + 10$

$a = 34$, $d = 32 - 34 = (-2)$, $a_n = 10$, $n = ?$

$a_n = a + (n-1)d$

$10 = 34 + (n-1) - 2$

$10 - 34 = (n-1) - 2$

$\frac{-24}{-2} = (n-1)$

$n = 12 + 1$

$n = 13$.

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{13}{2}(34 + 10)$

$S_n = \frac{13}{2} \times \cancel{44}^{22}$

$S_n = 286$

(iii) $(-5) + (-8) + (-11) \ldots + (-230)$

Here, $a = -5$, $a_n = (-230)$, $d = -8+5 = (-3)$, $n = ?$

~~By putting~~

$a_n = a + (n-1)d$

$-230 = -5 + (n-1) - 3$

$-230 + 5 = (n-1) - 3$

$\frac{-225}{-3} = (n-1)$

$75 = n - 1$

$n = 75 + 1$

$n = 76$

$S_n = \frac{n}{2}(a+l)$

$\Rightarrow S_n = \frac{76}{2}[(-5) + (-230)]$

$\Rightarrow S_n = 38(-235)$

$\Rightarrow S_n = (-8930)$.

$\overset{1}{2}\overset{1}{3}5$
$\times\ 38$
1880
$+705$
8930

(3) (i) Given =

$a=5,\ d=3,\ a_n=50,\ \text{find } n=?\text{ and } S_n=?$

$a_n = a+(n-1)d$

$50 = 5+(n-1)3$

$50-5 = (n-1)3$

$\frac{45}{3} = n-1$

$n = 15+1$

$n = 16$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_n = \frac{16}{2}[2\times5+(16-1)3$

$S_n = 8[10+45]$

$S_n = 8(55)$

$S_n = 440$.

(ii) Given =

$a=7,\ a_{13}=35,\ d=?,\ S_{13}=?$

$\cancel{a_{13} = a+12d}$ $\quad a_n = a+(n-1)d$

$\Rightarrow 35 = 7+(13-1)d$

$\Rightarrow 35-7 = 12d$

$\Rightarrow 28 = 12d$

$d = \frac{\cancel{28}\ \cancel{14}\ 7}{\cancel{12}\ \cancel{6}\ 3} = \frac{7}{3}$ $\quad \therefore \boxed{d=\frac{7}{3}}$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{13} = \frac{13}{2}(35+7)$$

$$S_{13} = \frac{13}{2} \times 24^{12}$$

$$\therefore S_{13} = 273$$

(iii) Here,

$a_{12} = 37$, $n = 12$, $d = 3$, $a = ?$ and $S_{12} = ?$

to find the value of 'a'.

$$a_n = a + (n-1)d$$

$$37 = a + (12-1) \times 3$$

$$37 = a + 33$$

$$a = 37 - 33$$

$$\boxed{\therefore a = 4}$$

$$S_{12} = \frac{n}{2}(a + l)$$

$$S_{12} = \frac{12}{2}(4 + 37)$$

$$S_{12} = \frac{12 \times 41}{2}$$

$$S_{12} = \frac{492}{2} \quad \therefore \boxed{S_{12} = 246}$$

(iv) Here, $a_3 = 15$, $S_{10} = 125$, $n = 10$, $d = ?$, $a_{10} = ?$

$a_3 = 15$

$a_{15} = a + 2d$ — (i)

$$S_n = \frac{n}{2}[2a + (10-1)d]$$

$125 = 5[2a + 9d]$

$\frac{125}{5} = 2a + 9d$

$25 = 2a + 9d$ — (ii)

To elemenate eq. (i) from (iii)

$\frac{1}{2}(25 = 2a + 9d)$

$\frac{25}{2} = \frac{2a}{2} + \frac{9}{2}d$

$12.5 = a + 4.5d$ — (iii)

Subtracting eq. (i) from (iii)

$$\begin{array}{r} a + 4.5d = 12.5 \\ (-)a \overset{+}{(-)} 2d = (-)15 \\ \hline 2.5d = -2.5 \end{array}$$

$d = \frac{-2.5}{2.5} = (-1)$.

$\therefore d = (-1)$

Putting the value of 'd' in eq. (i)

$\Rightarrow a_3 = a + 2d \Rightarrow 15 = a + 2 \times (-1) \Rightarrow a = 15 + 2$

$\boxed{\therefore a = 17}$

Here, $a = 17$, $d = (-1)$, $a_3 = 15$, $S_{10} = 125$, $a_{10} = ?$

$a_{10} = a + 9d$

$a_{10} = 17 + 9(-1)$

$a_{10} = 17 - 9$

$\boxed{\therefore a_{10} = 8}$

(v) Here,

$d = 5$, $S_9 = 75$, $n = 9$, $a_n = ?$, $a = ?$

$\cancel{S = \frac{n}{2}(a+l)}$ $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_9 = \frac{9}{2}[2a + \underbrace{(9-1)}_{8}5]$

$\Rightarrow 75 = \frac{9}{2}(2a+40)$

$\Rightarrow 75 = \frac{9}{2} \times 2[a+20]$

$\Rightarrow 75 = 3(a+20)$

$\Rightarrow 25 = 3a+60$

$\Rightarrow 25-60 = 3a$

$\Rightarrow a = \frac{-35}{3}$

We know that,

$a_n = a+(n-1)d$

$a_9 = a+(9-1)5$

$a_9 = \frac{-35}{3} + \frac{40 \times 3}{3}$

$a_9 = \frac{-35+120}{3}$ $\boxed{\therefore a_9 = \frac{85}{3}}$

(vi) $a=2$, $d=8$, $S_n=90$ to find n, a_n

$S_n = \frac{n}{2}[2a+(n-1)d]$

$90 = \frac{n}{2}[2\times2+(n-1)8]$

$\underbrace{90\times2}_{180} = n(8n-4)$

$4(2n^2-n-45=0)$

$\boxed{2n^2-n-45=0}$

$2x^2 - 10x - 9x - 45$

$2x(x-5) - 9(x-5)$

So, $x = 5$ & $x = \frac{9}{2}$

$\boxed{\therefore x = 5}$

$a_n = a + (n-1)d$

$a_n = 2 + (5-1)8$

$a_n = 2 + \underbrace{4 \times 8}_{32}$

$\boxed{\therefore a_n = 34}$

(vii) $a = 8$, $a_n = 62$, $S_n = 210$, $n = ?$ $d = ?$

$S_n = \frac{n}{2}(a + l)$

$210 = \frac{n}{2}(8 + 62)$

$210 = \frac{n}{\cancel{2}} \times \cancel{70}^{35}$

$\frac{\cancel{210}^{6}}{\cancel{35}} = n$

$\boxed{\therefore n = 6}$

$a_n = a + (n-1)d$

$62 = 8 + (6-1)d$

$62 = 8 + 5d$

$62 - 8 = 5d$

$\boxed{d = \frac{54}{5}}$

(viii) Here,
$a_n = 4$, $S_n = (-14)$, $n = ?$, $a = ?$, $d = 2$

$a_n = a + (n-1)d \Rightarrow 4 = a + (n-1)2$
$\Rightarrow 4 = a + 2n - 2 \Rightarrow a + 2n = 4 + 2$
$\Rightarrow a + 2n = 6$
$\Rightarrow a = 6 - 2n$ —(i)

$S_n = \frac{n}{2}(a + a_n)$
$\Rightarrow (-14) \times 2 = n(a + 4)$
$\Rightarrow -28 = n(6 - 2n + 4)$
$\Rightarrow -28 = n(-2n + 10)$
$\Rightarrow 2n^2 - 10n - 28 = 0$
$\Rightarrow 2(n^2 - 5n - 14 = 0)$

$n^2 - 5n - 14 = 0$ —(ii)
$\Rightarrow n^2 + 2n - 7n - 14$
$\Rightarrow n(n+2) - 7(n+2)$
$\Rightarrow (n+2)(n-7)$
$\therefore n = (-2); n = 7$.

As 'n' can't be (-ve) so, $n = 7$.
Putting the value of 'n' in eq.-(i)

$a = 6 - 2n$
$a = 6 - 7 \times 2$
$a = 6 - 14$

$\boxed{\therefore a = (-8)}$

(ix) Here,

$a = 3,\ n = 8,\ s = 192,\ d = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 192 = \frac{8}{2}[2 \times 3 + (8-1)d]$

$\Rightarrow 192 = 4(6 + 7d)$

$\Rightarrow \frac{192}{4} = 6 + 7d$

$\Rightarrow d = \frac{42}{7}$

$\boxed{\therefore d = 6}$

(x) $l = 28,\ s = 144,\ n = 9,\ a = ?$

$S_n = \frac{n}{2}(a + l)$

$\Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow 144 \times \frac{2}{9} = a + 28$

$\Rightarrow \frac{288}{9} = a + 28$

$\Rightarrow a = 32 - 28$

$\boxed{\therefore a = 4}$