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AP - 9, 17, 25, ...

$$a = 9, d = 17 - 9 = 8, S_n = 636,$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 636 = \frac{n}{2} [2 \times 9 + (n-1)8]$$

$$\Rightarrow 636 = \frac{n}{2} [18 + 8n - 8]$$

$$\Rightarrow 636 = \frac{n}{2} (9 + 4n - 4)$$

$$\Rightarrow n(4n+5)$$

$$\boxed{4n^2 + 5n - 636 = 0}$$

$$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow n(4n+53) - 12(4n+53) = 0$$

$$\Rightarrow (4n+53)(n-12) = 0$$

$$n = \frac{-53}{4} \text{ or } n = 12$$

Therefore, $n = 12$.

5) Given =

$$a = 5, d = 45, S_n = 400, n = ?$$

$$S_n = \frac{n}{2} (a + d)$$

$$400 = \frac{n}{2} (5 + 45)$$

$$800 = n(50)$$

$$n = \frac{800}{50} = 16$$

$$\therefore \boxed{n = 16}$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 400 = \frac{16}{2} [2 \times 5 + (16-1)d]$$

$$\Rightarrow 400 \times 2 = 16 [10 + 15d]$$

$$\Rightarrow \frac{800}{16} = 10 + 15d$$

$$\Rightarrow 50 - 10 = 15d$$

$$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Here,

⑥ $a = 17, a_n = 350, d = 9, n = ?, S_n = ?$

$$a_n = a + (n-1)d$$

$$\Rightarrow 350 = 17 + (n-1)9$$

$$\Rightarrow 350 - 17 = (n-1)9$$

$$\Rightarrow \frac{333}{9} = (n-1)$$

$$\Rightarrow n = 37 + 1$$

$$\Rightarrow n = 38$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow S_n = \frac{38}{2} [2 \times 17 + (38-1)9]$$

$$\Rightarrow S_n = 19 (34 + 353)$$

$$\Rightarrow S_n = 19 \times 367 = 6973$$

Hence,
 $(7) \quad n=22, d=7, l=149,$

we know that,

$$22^{\text{nd}} \text{ term is } 149.$$

$$\text{Hence, } a + 21d = 149 \text{ --- (i)}$$

$$a_{22} = a + (n-1)d$$

$$= (a + (22-1)d)$$

Putting the value of 'd' in eq. (i.)

$$a + 21 \times 7 = 149$$

$$a = 149 - 147$$

$$a = 2$$

we can find the sum of 22 terms by the formula.

$$S_n = \frac{n}{2} (n + l)$$

$$S_n = \frac{22}{2} (2 + 149)$$

$$S_n = 11 (151)$$

$$\therefore S_n = 1661$$

So, the sum of the 1st to 22nd term AP is 1661.

8) $a_n = 51$, $d = 18 - 14 = 4$, $a = 10$, $S_n = ?$

2nd term of AP - 14

$$a + d = 14$$

$$\Rightarrow a + 4 = 14$$

$$a = 14 - 4$$

$$\boxed{a = 10}$$

$$a_n = a + (n-1)d$$

$$a_n = 10 + (5-1)4$$

$$a_n = 10 + 5 \times 4$$

$$a_n = 210 \text{ } \cancel{200} = 1$$

$$S_n = \frac{n}{2} [a + a_n]$$

$$S_n = \frac{n}{2} [a + l]$$

$$S_n = \frac{51}{2} [10 + 210]$$

$$S_n = \frac{51}{2} \overset{110}{\cancel{220}} = 51 \times 110$$

$$S_n = 5610$$

So, $S_{51} = 5610$.

9)

$$S_7 = 49$$

$$S_{17} = 289$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_7 = \frac{7}{2} [2a + (7-1)d]$$

$$49 = \frac{7}{2} (2a + 6d)$$

$$7 = (a + 3d)$$

$$a + 3d = 7 \text{ --- (i)}$$

Similarly, $S_{17} = \frac{17}{2} [2a + (17-1)d]$

$$289 = \frac{17}{2} [2a + 16d]$$

$$17 = (a + 8d)$$

$$a + 8d = 17 \text{ --- (ii)}$$

Subtracting eq. (i) from eq. (ii)

$$5d = 10$$

$$d = \frac{10}{5} = 2$$

From eq. (i)

$$a + 3(2) = 7$$

$$a + 6 = 7$$

$$a = 7 - 6 = 1$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(1) + (n-1)2]$$

$$= \frac{n}{2} (2 + 2n - 2)$$

$$= \frac{n}{2} \times 2n = n^2$$

(10)

$$a_n = 3 + 4n$$

$$a_1 = 3 + 4(1) = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$a_3 = 3 + 4(3) = 3 + 12 = 15$$

$$a_4 = 3 + 4(4) = 3 + 16 = 19$$

It can be observed that,

$$a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4$$

$$a_4 - a_3 = 19 - 15 = 4$$

i.e., $a_{k+1} - a_k$ is same every time.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{13} = \frac{13}{2} [2(7) + (13-1)4]$$

$$= \frac{13}{2} [(14) + (56)]$$

$$= \frac{13}{2} \times 70 = 525$$

(11)

$$S_n = 4n - n^2$$

$$S_1 = 4 - 1 = 3$$

$$S_2 = (4 \times 2) - (2)^2 = 4$$

$$S_3 = (4 \times 3) - (3)^2 = 3$$

$$\therefore a_3 = S_3 - S_2 = (-1)$$

$$\therefore a_{10} = S_{10} - S_9 = (-15)$$

$$\therefore a_n = S_n - S_{n-1}$$

$$a_n = 5 - 2n$$

35
x 15
525
35
x 13
525
15
x 8
120
14
x 8
112

(12) The multiples of 6 are -

$$a = 6$$

$$d = 6$$

$$S_{40} = ?$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{40} = \frac{40}{2} [2(6) + (40-1)6]$$

$$= 20 (12 + 234)$$

$$= 20 \times 246$$

$$= 4920$$

(13) We can get the AP as -
8, 16, 24, ... 120

$$a = 8, d = 8, a_n = 120, n = 15$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{15}{2} [2 \times 8 + 14 \times 8]$$

$$S_n = \frac{15}{2} \times 128$$

$$S_n = 960$$

$$\therefore S_n = 960$$

(14) The sum of odd no.'s between 0 and 50 can be represented by A.P. as

$$1, 3, 5, \dots, 49$$

Here, $a=1$, $d=2$, $a_n=49$, $n=?$

$$\begin{aligned} a_2 - a_1 &= 3 - 1 = 2 \\ a_3 - a_2 &= 5 - 3 = 2 \end{aligned}$$

$$a_n = a + (n-1)d$$

$$49 = 1 + (n-1)2$$

$$49 - 1 = (n-1)2$$

$$\frac{48}{2} = n-1 \Rightarrow n = \frac{48}{2} + 1$$

$$n = \frac{49}{2}$$

$$a_n = a + (n-1)d$$

$$49 = 1 + (n-1)2$$

$$\frac{48}{2} = n-1$$

$$n = 24 + 1$$

$$\therefore n = 25$$

$$S_n = \frac{n}{2}(a+d)$$

$$S_n = \frac{25}{2}(1+49)$$

$$S_n = \frac{25}{2} \times 50$$

$$S_n = 25 \times 25, \text{ which is}$$

$$\therefore S_n = 625$$

(15) The penalty for delay in completion of the construction work is = ₹200 for 1st day.

The penalty given by construction workers due to delay in 2nd day = ₹250

The penalty given for construction work due to day in 3rd day = ₹300.

The penalty in each succeeding day is more than the preceding day by = ₹50

The amount of money paid for delaying 30 days =

A/Q

Here, $a = 200$, $d = 50$, $n = 30$, $S_n = ?$

$$S_{30} = \frac{30}{2} [2 \times 200 + (30 - 1) 50]$$

$$S_{30} = 15 [400 + (29 \times 50)]$$

$$S_{30} = 15 [400 + 1450] \Rightarrow S_{30} = 15 \times 1850$$

$$S_{30} = 15 \times ₹27,750$$

So, the sum of amount paid by the workers for delaying of 30 days is = ₹27,750.

(16) The sum of amount of money for prizes (Sn) = ₹ 700.

The no. of cash prizes to be given to students (n) = 7

Each prize is 20 less than the preceding, so here $d = (-20)$

The value of each prizes =

Given that $S_7 = 700$

$$\Rightarrow \frac{7}{2} [2a + (7-1)d] = 700$$

$$\Rightarrow [2a + (6)(-20)] = 100$$

$$\Rightarrow a + 3(-20) = 100$$

$$\Rightarrow a - 60 = 100$$

$$\Rightarrow a = 160$$

So, the AP is - 160, 140, 120, 100, 80, 60, 40.

Therefore the value of each of prizes was - ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60, ₹ 40

(17) It can be observed that the number of trees planted by the students is in a A.P.

1, 2, 3, 4, 5, ... 12.

$a = 1$, $d = 2 - 1 = 1$, $S_n = ?$, $n = 12$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{12} = \frac{12}{2} [2(1) + (12-1)(1)]$$

$$S_{12} = 6(2+11)$$

$$\textcircled{1} S_{12} = 6(13)$$

$$\therefore S_{12} = 78$$

Therefore, no. of trees planted by 1 section of the classes = 78.

No. of trees planted by 3 sections of the classes = 3×78
 $= 234$.

Therefore, ~~no. of~~ 234 trees will be planted by the students.

18) Semi-perimeter of circle = πr
 $d_1 = \pi(0.5) = \frac{\pi}{2}$ cm.

$$d_2 = \pi(1) = \pi$$
 cm

$$d_3 = \pi(1.5) = \frac{3\pi}{2}$$
 cm.

So, AP - $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \dots$

$$a = \frac{\pi}{2}, d = \pi - \frac{\pi}{2} = \frac{\pi}{2}, n = 13, S_{13} = ?$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{13} = \frac{13}{2} \left[2 \left(\frac{\pi}{2} \right) + (13-1) \left(\frac{\pi}{2} \right) \right]$$

$$= \frac{13}{2} \left[\pi + \frac{12\pi}{2} \right]$$

$$= \left(\frac{13}{2} \right) (7\pi)$$

$$= \frac{91\pi}{2} = \frac{13 \times 11}{2 \times 7} = 13 \times 11 = 143$$

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

(19) It can be observed that the no. of logs in rows are in an A.P.

$$20, 19, 18, \dots$$

$$a_1 = 20, d = (-1), S_n = ?$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 200 = \frac{n}{2} [2(20) + (n-1)(-1)]$$

$$\Rightarrow 400 = n(40 - n + 1)$$

$$\Rightarrow 400 = n(41 - n)$$

$$\Rightarrow 400 = 41n - n^2 \Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25 + 400 = 0$$

$$\Rightarrow n(n-16) - 25(n-16) = 0$$

$$\Rightarrow (n-16)(n-25) = 0$$

$$a_n = a + (n-1)d$$

$$a_{16} = 20 + (16-1)(-1)$$

$$a_{16} = 20 - 15$$

$$a_{16} = 5$$

Similarly,

$$a_{25} = 20 + (25-1)(-1)$$

$$a_{25} = 20 - 24$$

$$a_{25} = -4$$

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

20) The distance of potatoes are as follows:-
5, 8, 11, 14, ...

It can be observed that these distances are in A.P.

$$a = 5, d = 8 - 5 = 3$$

~~$$d = 8 - 5 = 3$$~~

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(5) + (10-1)3] = 5[10 + 27]$$

$$S_{10} = 5(10 + 27) = 5(37) = 185$$

∴ Therefore, total distance that the competitor will run = 2×185
= 370 m.