$$\frac{BM}{AB} = \frac{AN}{AD}$$

Hence proved.

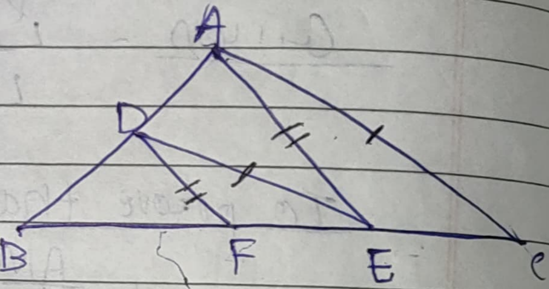
(4) <u>Given that -</u>

DE || AC, &

DF || AE

<u>To prove that -</u>

$$\frac{BF}{FE} = \frac{BE}{EC}$$



In $\triangle$ ABE, as DF || AE

$$\frac{BD}{AD} = \frac{BE}{BF}$$ (basic proportionality theorem) —— (1)

In $\triangle$ ABC, as DE || AC

$$\frac{BD}{AD} = \frac{EC}{BE}$$ (basic proportionality theorem) —— (11)

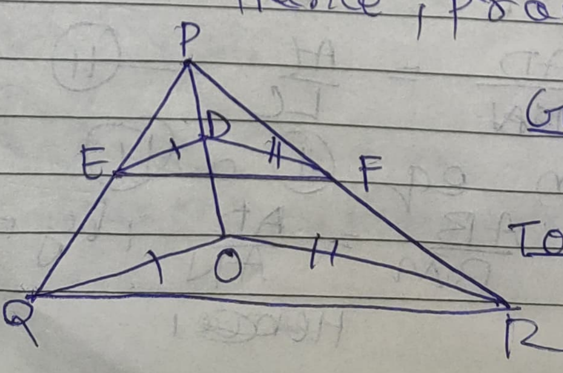
From eq. (1) and (2)

$$\frac{BD}{AD} = \frac{EF}{BF} = \frac{EC}{BE}$$

$$\therefore \frac{BF}{FE} = \frac{BE}{EC}$$

Hence, proved.

(5)



<u>Given</u> - DE || OQ

DF || OR

<u>To show that</u> -

EF || QR

In Δ PEQ

$\frac{PE}{PQ} = \frac{PD}{DO}$ (basic proportionality theorem) — (1)

In Δ POR

$\frac{PF}{FR} = \frac{PD}{DO}$ (by BPT) — (2)

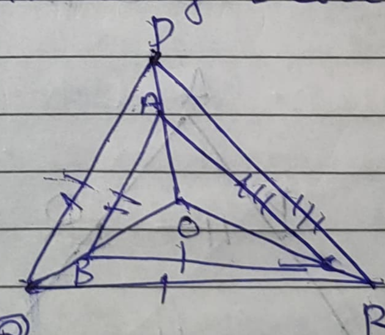
From eq - (1) and (2)

$\frac{PE}{PQ} = \frac{PF}{FR}$

As the sides are proportional then then;

EF // QR (By converse of basic proportionality theorem)

(5)



Given - A, B, C are point on sides OP, OQ and OR respectively.

AB // PQ, &

AC // PR

To show that - BC // QR

In Δ POQ

$\frac{OA}{AP} = \frac{OB}{BQ}$ (By BPT) — (1)

In Δ POR

$\frac{OA}{AP} = \frac{OC}{CR}$ (By BPT) — (2)

From Eq. (1) and (2)

$\frac{OB}{BQ} = \frac{OC}{CR}$

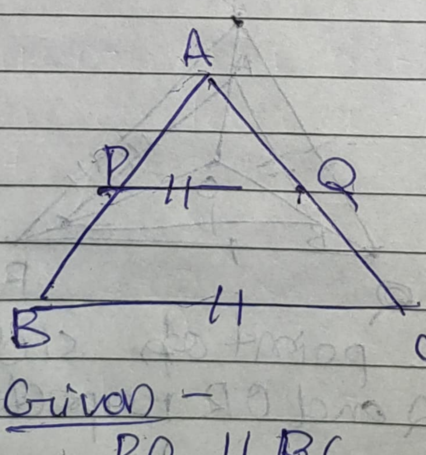
Therefore, by reciprocating,

$\frac{OB}{QB} = \frac{OC}{RC}$

Considering this in Δ ORQ

BC $\parallel$ QR [converse basic proportionality theorem]

(7)



Given -

PQ $\parallel$ BC

In Δ ABC

$\frac{AQ}{QC} = \frac{AP}{PB}$ (As P is the mid-point of AB)

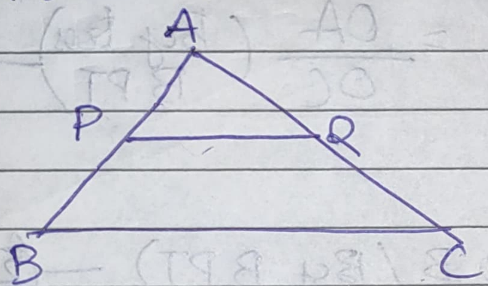
So, AP = PB

$\frac{AQ}{QC} = \frac{\cancel{AP}^1}{\cancel{PB}_1}$; $\frac{AQ}{(by BPT)} \stackrel{1\times}{=} QC$

since, AQ = QC

Hence, Q is the mid-point of AC.

(8)



~~Given~~ $\frac{AP}{AB} = \frac{AQ}{QC}$

In ΔABC

~~AQ, P~~ As P and Q are mid points of sides AB and AC.

Thus, $\frac{AD}{AB}$ is proportional to $\frac{AQ}{QC}$

(~~By mid-p~~

So, $\frac{AB}{AP} = \frac{AC}{AQ}$

Therefore, PQ ∥ BC (By converse of basic proportionality theorem)

(9) Given – AB ∥ DC.

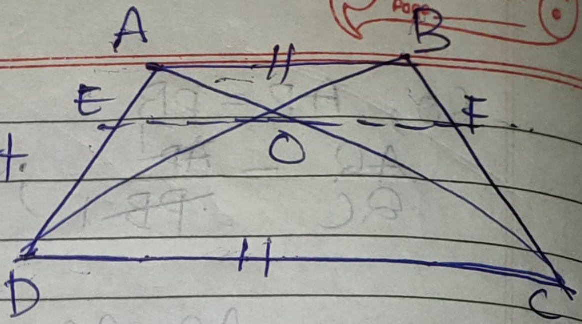
diagonals intersect at O.

To show that – $\frac{AO}{BO} = \frac{CO}{DO}$

~~In Δ ADC~~

Draw EF such that

EF ∥ DC ∥ AB



In Δ ADC

$\cancel{\frac{AE}{AD} = \frac{AO}{OC}}$ $\frac{EA}{ED} = \frac{OA}{OC}$ (By BPT) — (1)

In Δ BAD

$\frac{EA}{ED} = \frac{OB}{OD}$ (By BPT) — (2)

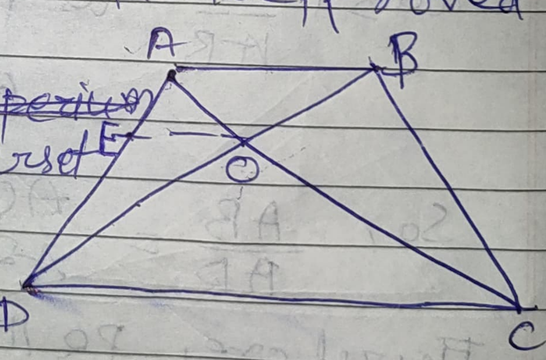
From eq. (1) and (2)

$\frac{OA}{OC} = \frac{OB}{OD}$

~~By reciprocating~~ Therefore, $\frac{AO}{BO} = \frac{CO}{DO}$

Hence, Proved.

(10) Given ABCD is a ~~trapezium~~ quadrilateral intersect at point O.



$\frac{AO}{BO} = \frac{CO}{DO}$

To show that:

ABCD is a trapezium or AB ∥ DC.

~~In Δ ADB~~

Draw EO such that ~~EO~~ ∥ DC ∥ AB

In ΔADB,

$\frac{AE}{ED} = \frac{BO}{DO}$ (By basic proportionality theorem)

So, $EO \parallel$ ~~DC~~ AB — (By converse of BPT) — ①

In ΔADC

$\frac{AE}{ED} = \frac{AO}{OC}$ (By basic proportionality theorem)

So, $EO \parallel DC$ (By converse of BPT) — ②

That is, $EO \parallel AB \parallel DC$

Hence, $AB \parallel DC$

Therefore quadrilateral ABCD is a trapezium