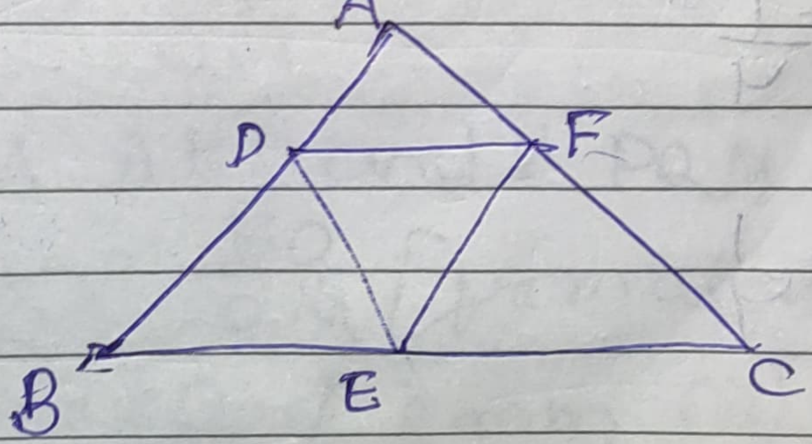
(5)



DF || BC, ∴ as D and F is the median of sides AB and AC.

$DF = 1/2\ BC$
$BE = 1/2\ BC$
$2(BE)\ \ BC$

(i) considering,
ΔDBE and ΔABC
$\angle DBE = \angle ABC$
$\angle DEB = \angle ACB$
So, by AA similarity criterion
$\Delta DBE \sim \Delta ABC$

$\frac{DB}{AD} = \frac{BE}{BC} = \frac{FC}{AC}$ (corresponding sides of similar are equal)

$\frac{ar(\Delta DBE)}{ar(\Delta ABC)} = \left(\frac{BE}{BC}\right)^2$

$\frac{ar(DBF)}{ar(ABC)} = \left(\frac{BE}{2(BE)}\right)^2$

$\frac{ar(DBC)}{ar(ABC)} = \frac{\cancel{(BE)^2}}{4\cancel{(BE)^2}}$

(i) Hence, $\frac{ar(DBE)}{ar(ABC)} = \frac{1}{4}$

$ar(DBE) = \frac{1}{4} ar(ABC)$

(ii) $\frac{ar(FEC)}{ar(ABC)} = \frac{1}{4}$

(iii) $\frac{ar(ADF)}{ar(ABC)} = \frac{1}{4}$

Date
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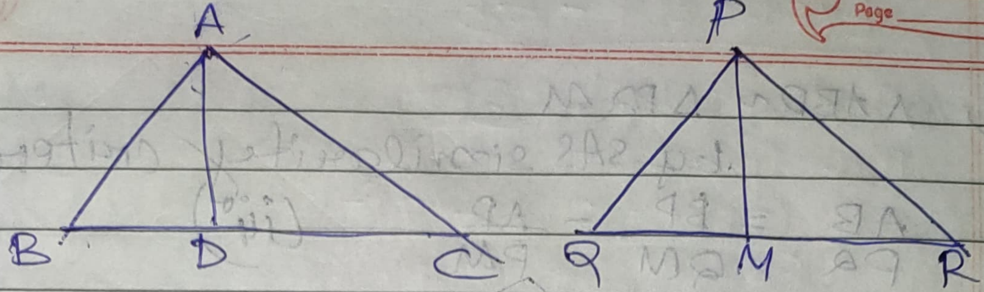
(6)



Given - $\Delta ABC \sim \Delta PQR$

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ —— (i) [corresponding parts of similar Δ's are proportional]

$\angle A = \angle P$; $\angle B = \angle Q$; $\angle C = \angle R$ —— (ii)

we know that,

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

As, AD and PM are the medians of ΔABC and ΔPQR, then,

$BD = 1/2\, BC$
$QM = 1/2\, QR$

we can write eqⁿ (i) as,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{1/2\, BC}{1/2\, QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

In Δ ABD and Δ PQM

$\frac{AB}{PQ} = \frac{BD}{QM}$ [from above]

$\angle B = \angle Q$ [from (ii)]

$\therefore \triangle ABD \sim \triangle PQM$

by SAS similarity criterion

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad \text{(iii)}$$

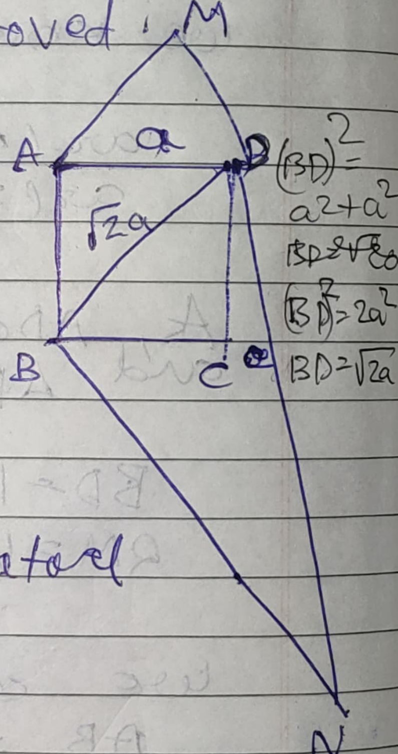
From eq. (i) and (iii), we ~~g~~ can say;

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\text{So, } \frac{ar(ABC)}{ar(PQR)} = \left(\frac{AD}{PM}\right)^2$$

Hence proved.

(7) $ar(NAB) = \frac{1}{2} ar(DBN)$



consider side of the square = a,

diagonal of the square = $\sqrt{2}a$

Equilateral $\triangle AMB$ lies on the one side of the square

Let the sides of the equilateral $\triangle$ be $x = a$

Similarly,

Equilateral $\triangle DBN$ lies on the one side of the square,

Let the side of the $\triangle DBN$, $y = \sqrt{2}a$.

As we know, all equilateral $\triangle$ have angle,

That is 60° each (sides are equal and proportional)

So, by AAA similarity criterion

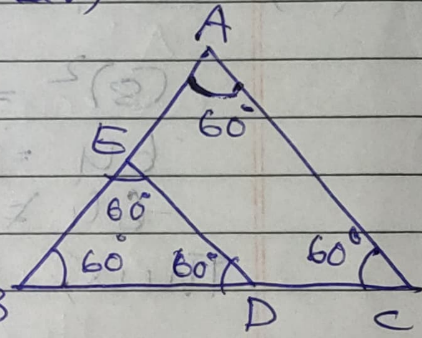
$\Delta AMB \sim \Delta DBN$

$$\frac{ar(AMB)}{ar(DBN)} = \left(\frac{x}{y}\right)^2 = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{a^2}{2a^2} = \frac{1}{2}$$

Therefore,

$ar(AMB) = \frac{1}{2} ar(DBN)$

(8) Let side of $\Delta ABC = x$



Therefore, side of $\Delta BDE = \frac{x}{2}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{x^2}{\frac{x^2}{2}} = \frac{4}{1}$$

Hence (c) 4:1

(9) It is given that the sides are in the ratio 4:9

Therefore, ratio between areas of these triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, (D) 16:81.