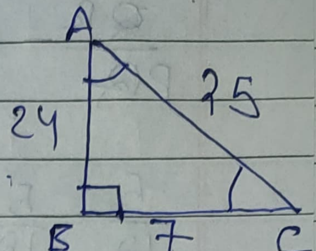
INTRODUCTION TO TRIGONOMETRY

EXERCISE - 8.1

① ABC is a Δ right-angled at B.



AC

By pythagoras theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (24)^2 + (7)^2$

$(AC)^2 = 576 + 49$

$(AC)^2 = 625$

$AC = \sqrt{625} = 25$

$24 \times 24 = 96 + 480 = 576$

(i) sin A, cos A

$\sin A = \frac{BC}{AC} = \frac{7}{25}$

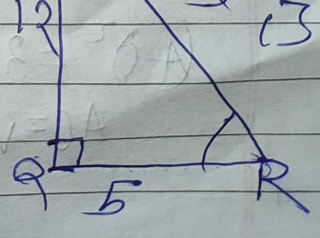
$\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) sin C, cos C

$\sin C = \frac{AB}{AC} = \frac{24}{25}$

$\cos C = \frac{BC}{AC} = \frac{7}{25}$

② By pythagoras theorem,



$(QR)^2 = (PR)^2 - (PQ)^2$

$QR^2 = 169 - 144$

$QR = \sqrt{25} = 5$

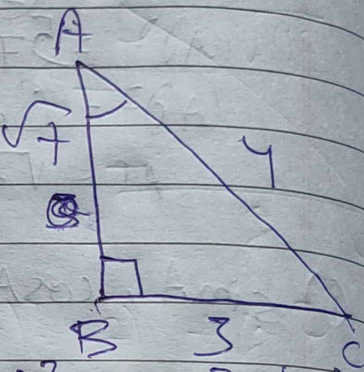
$\tan P - \cot R$

$\frac{QR}{PQ} - \frac{QR}{PQ}$, which is

$\frac{5}{12} - \frac{5}{12} = 0.$

Therefore, $\tan P - \cot R = 0.$

(3) $\sin A = \frac{3}{4} = \frac{p}{h}$



$\cos A = \frac{\sqrt{7}}{4}$

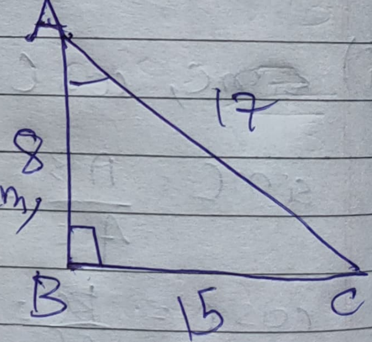
$\tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$

$(AB)^2 = (4)^2 - (3)^2$

$(AB)^2 = 16 - 9$

$AB = \sqrt{7}$

(4) $15 \cot A = 8$ [Given]



$\cot A = \frac{8}{15} = \frac{b}{p}$

By pythagoras theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (8)^2 + (15)^2$

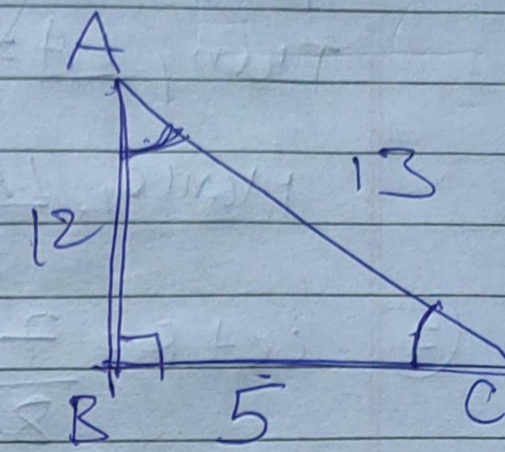
$(AC)^2 = 225 + 64$

$AC = \sqrt{289} = 17$

$$\sin A = \frac{BC}{AC} = \frac{15}{17}$$

$$\sec A = \frac{AC}{AB} = \frac{17}{8}$$

(5) $$\sec\theta = \frac{13}{12} = \frac{h}{b}$$



$$\sin\theta = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos\theta = \frac{AB}{AC} = \frac{12}{13}$$

$$(BC)^2 = (13)^2 - (12)^2$$
$$(BC)^2 = 169 - 144$$
$$BC = \sqrt{25} = 5$$

$$\tan\theta = \frac{BC}{AB} = \frac{5}{12}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13}{5}$$

$$\cot\theta = \frac{AB}{BC} = \frac{12}{5}$$

From eq. (1) and (2).

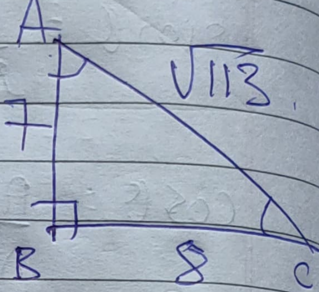
we get, $AB = BC$.

As, $AB = BC$, it's an isosceles $\triangle$.

Then, $\angle A = \angle B$ (~~sides~~ angle opp. to equal sides)

Hence, $\angle A = \angle B$.

(7) $\cot\theta = \frac{7}{8} = \frac{b}{p}$



(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\frac{1-\sin^2\theta}{1-\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\left(\frac{8}{\sqrt{113}}\right)^2}{\left(\frac{7}{\sqrt{113}}\right)^2}$

$= \frac{64}{113} \times \frac{113}{49}$

$= \frac{64}{49}$

$(AC)^2 = (7)^2 + (8)^2$

$(AC)^2 = 49 + 64$

$(AC)^2 = \sqrt{113}$

$\sin\theta = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{7}{\sqrt{113}}$

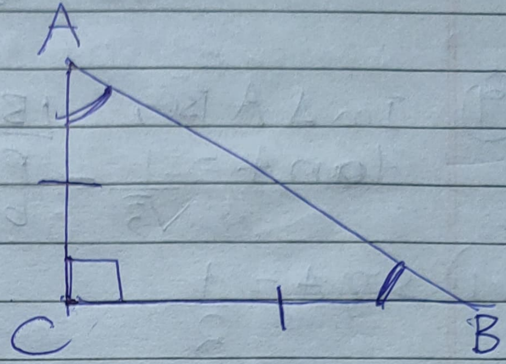
$\frac{1-\sin^2\theta}{1-\cos^2\theta}$ $\left[a^2+b^2 = (a+b)(a-b)\right]$

$\frac{\cos^2\theta}{\sin^2\theta} = \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{49}{113} \times \frac{113}{64} = \frac{49}{64}$

(ii) $\cot^2\theta$

Here, $\cot\theta = \frac{b}{p} = \frac{AB}{BC} = \frac{7}{8}$

$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$.



(6) $\cos A = \cos B$

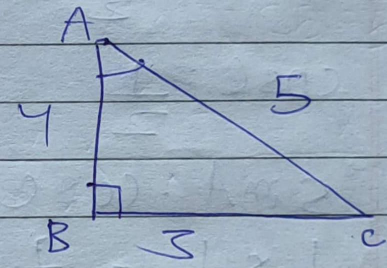
$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

$\Rightarrow \angle A = \angle B$ (Proved)

(8) $3\cot A = 4$

$\cot A = \frac{4}{3} = \frac{b}{p}$



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\tan A = \frac{3}{4}$

$(AC)^2 = (AB)^2 + (BC)^2$

$AC^2 = (4)^2 + (3)^2$

$AC^2 = 16 + 9$

$AC = \sqrt{25} = 5$

$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}}$

$= \frac{7}{\cancel{16}} \times \frac{\cancel{16}}{25} = \frac{7}{25}$

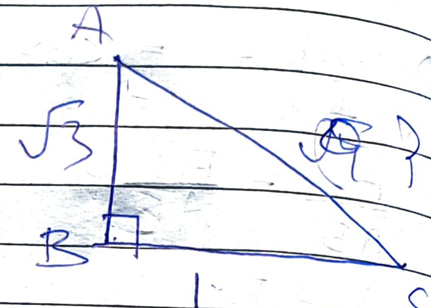
$RHS = \cos^2 A - \sin^2 A$

$\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25}$

$= \frac{16-9}{25} = \frac{7}{25}$

(9) In $\triangle ABC$, $\angle B = 90^\circ$

$\tan A = \frac{1}{\sqrt{3}} = \frac{P}{b}$



$(AC)^2 = (\sqrt{3})^2 + (1)^2$

$AC = \sqrt{3+1}$

$AC = \sqrt{4} = 2$

(i) $\sin A = \frac{1}{2}$

$\cos A = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\sqrt{3}}{2}$

$\cos C = \frac{1}{2}$

$\sin A \cdot \cos C + \cos A \cdot \sin C$

$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cdot \cos C - \sin A \cdot \sin C$

$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

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(10) In ΔPQR

$PR + QR = 25$ cm — (i)
$PR = 25 - QR$

So, by pythagoras theorem,
$PR^2 = PQ^2 + QR^2$
$625 - 25 = 50QR$
$QR = \frac{600}{50}$
$QR = 10$

Substituting the value of QR in eq-(i)
$PR = 25 - 12$
$PR = 13$ cm

$\sin P = \frac{12}{13}$
$\cos P = \frac{5}{13}$
$\tan P = \frac{12}{5}$

(11) (i) The value of tan A is always less than 1 False

Reason - The values of sinA or cosA is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of $\angle A$. True

(iii) cos A is the abbreviation used for the cosec of $\angle A$ False.
Reason - cosine A.

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(iv) cot A is the product of cot and A. False. Reason - Because, cot A is used as an abbreviation for the cotangent of $\angle A$ not the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some $\angle\theta$. False.

Reason - P, P < h, or it does not for 90. So, θ is an angles of $\frac{4}{3}$

EXERCISE - 8.2

(1)

(i) $\sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 60^\circ$

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2 \times 1 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \csc 30^\circ}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{(2+2\sqrt{3})}$