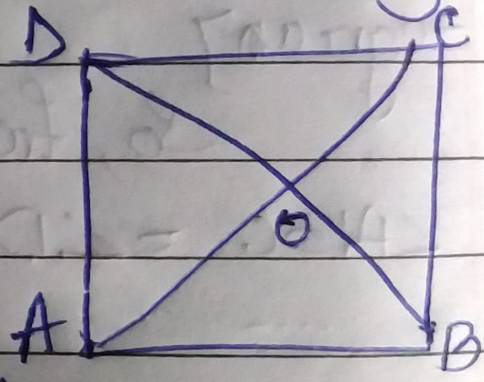
Q4. Show that diagonals of a square are equal and bisect each other at right angles.

ans) Let ABCD be a square such that its diagonal AC and BD intersected at O



(i) To prove that the diagonals are equal, we need prove AC = BD

In ΔABC and ΔBAD, we have

$AB = AB$ (common)

$BC = AD$ (side of a square ABCD)

$\angle ABC = \angle BAD$ [each angles $90°$]

$\therefore \Delta ABC \cong \Delta BAD$ (By SAS congruency)

$AC = BD$ (By CPCT) $\longrightarrow$ (i)

ii) $AD \parallel BC$ and AC is a transversal.

~~$\angle 1 = \angle 3$~~ $\angle 1 = \angle 3$ (alternate interior angles are equal)

$\angle 2 = \angle 4$

In ΔOAD and ΔOCB we have

$AD = CB$ (sides of a square ABCD)

$\left.\begin{array}{l}\angle 1 = \angle 3 \\ \angle 2 = \angle 4\end{array}\right\}$ proved

$\therefore \Delta OAD = \Delta OCB$ (By ASA)

$\Rightarrow OA = OC$ and $OD = OB$ (By CPCT)

the diagonals of a AC and BD bisect each other at O $\longrightarrow$ (ii)

iii) In ΔOBA and ΔODA we have

$OB = OD$ (proved)

$BA = DA$ [sides of a square ABCD]

$OA = OA$ = (common)

$\therefore \Delta OBA \cong \Delta ODA$ (By SSS congruency)

$\Rightarrow \angle AOB = \angle AOD$ [By CPCT] $\longrightarrow$ (iii)

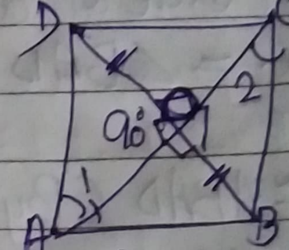
$\therefore \angle AOB$ and $\angle AOD$ ~~form~~ (linear pair)

$\angle AOB + \angle AOD = 180°$

$\angle AOB = \angle AOD = 90$ (By equation iii)

$\Rightarrow AC \perp BD \longrightarrow$ (iv)

Q5) ans) Let ABCD be a quadiletral such that diagonals AC and BD are equal and bisect each other at right angles.



Now, in ΔAOD and ΔAOB we have $\angle AOD = \angle AOB = 9$

$AO = AO$ (common)

$OD = OB$ [O is the midpoint of BD]

$\therefore \Delta AOD \cong \Delta AOB$ [By SAS congruency]

$\Rightarrow AD = AB$ [By CPCT] $\rightarrow$ (i)

we have,

$AB = BC \rightarrow$ (ii)

$BC = CD \rightarrow$ (iii)

$CD = DA \rightarrow$ (iv)

from equation (i), (ii), (iii), (iv) we have,

$AB = BC = CD = DA$

$\therefore$ Quadreiletral ABCD have all sides equal.

In ΔAOD and ΔCOB, we have

$AO = CO$ (given)

$OD = OB$ (given)

$\angle AOD = \angle COB$ (vertically opposite angle)

So, $\Delta AOD \cong \Delta COB$ (By SAS congruency)

$\therefore \angle 1 = \angle 2$ (By CPCT)

But, they form a pair of alternet interior angles.

$\therefore AD \parallel BC$

$AB \parallel DC$

$\therefore$ ABCD is parallelogram.

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Now, in ABC and BAD we have,

$AC = BD$ (Given)

$BC = AD$ (Proved)

$AB = AB$ (common)

$\Delta ABC \cong \Delta BAD$ (By SSS congruency)

$\angle ABC = \angle BAD$ (By CPCT) $\rightarrow$ (v)

$AD \parallel BC$ and AB is a transversal.

$\angle ABC + \angle BAD = 180^\circ \rightarrow$ (vi) (co-interior angle

$\Rightarrow \angle ABC = \angle BAD = 90^\circ$ (By equation (v) and (vi))

So, rhombus ABCD is having one angle equal to 90°

Thus, ABCD is a square.