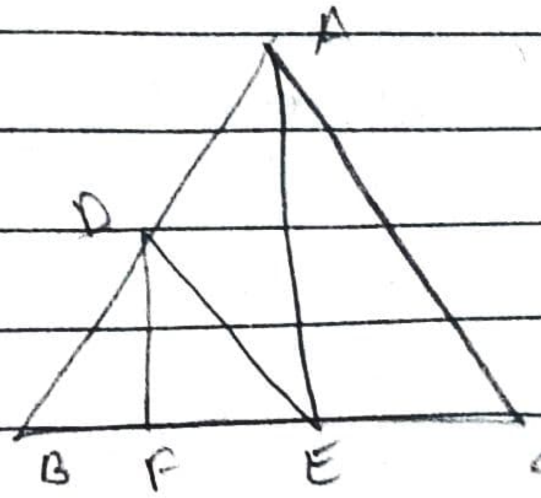
4. In ~~ΔAB~~ ΔBAC, $DE \parallel AC$ (given)

$\therefore \dfrac{BE}{EC} = \dfrac{BD}{AD}$ —— (1) (By B.P.T)



Similarly, in ΔBAE, $DF \parallel AE$

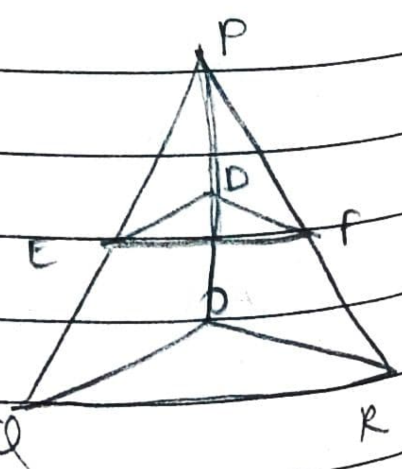
$\therefore \frac{BF}{FE} = \frac{BD}{DA}$ —— (ii) (By B.P.T)

from equation (i) and (ii), we get

$\frac{BE}{EC} = \frac{BF}{FE}$ (Hence proved)

5. In ΔPOQ, $DE \parallel OQ$

$\frac{PE}{EQ} = \frac{PD}{DO}$ —— (i) (By BPT)



In ΔPOR, $DF \parallel OR$

$\frac{PF}{FR} = \frac{PD}{DO}$ —— (ii) (By BPT)

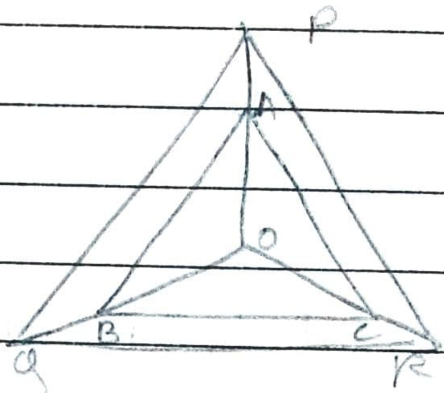
from eq^n (i) and (ii), we get

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$

6. Given, $AB \parallel PQ$

$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ}$ —— (i) (By BPT)



and $AC \parallel PQ$ (given)

$\Rightarrow \frac{OA}{AP} = \frac{OC}{CR}$ —— (ii) (By BPT)

from eq^n (i) and (ii), we get,

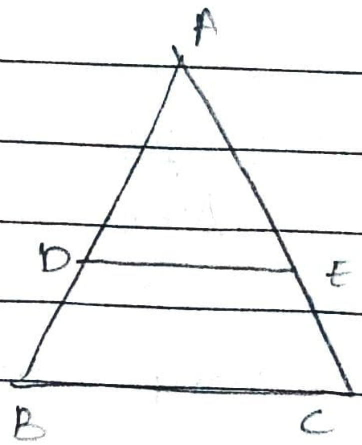
$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore$ $BC \parallel QR$ (By Converse BPT)

Hence, proved

7. Given

In ΔABC in which D is the mid-point of AB and $DE \parallel BC$.



To prove

$AE = EC$

Proof

In ΔABC, $DE \parallel BC$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ (By B.P.T)

But $AD = DB$

$\Rightarrow \frac{AD}{DB} = 1$

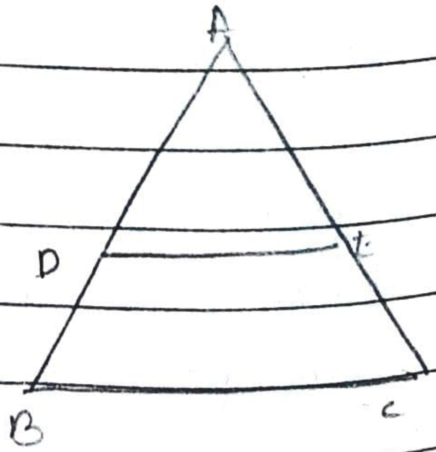
$1 = \frac{AE}{EC}$

$\Rightarrow AE = EC$

$\therefore$ Hence, DE bisects AC

8. Given

ΔABC in which D and E are the mid points of AB and AC respectively.



To prove

$DE \parallel BC$

Proof

D is the mid points of AB,

$AD = AB$

E is the mid point of AC,

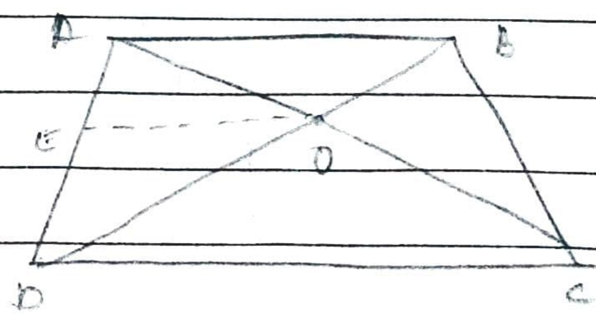
$AE = EC$

$\Rightarrow \cancel{\frac{AD}{DB}} \quad \cancel{\frac{AE}{EC}}$

$\frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow DE \parallel BC$ (by converse of BPT)



9. <u>Given</u>

ABCD is a trapezium in which $AB \parallel DC$ and diagonals intersect each other at O.

<u>To prove</u>

$\frac{AO}{BO} = \frac{CO}{DO}$

~~Proof~~

~~In Δ~~

Construction

Draw a line EO parallel to DC.

Proof

In Δ ADC,

EO $\parallel$ DC

$\Rightarrow \frac{AE}{ED} = \frac{AO}{CO}$ (By BPT) —— (i)

In Δ ABD,

EO $\parallel$ DC

but DC $\parallel$ AB

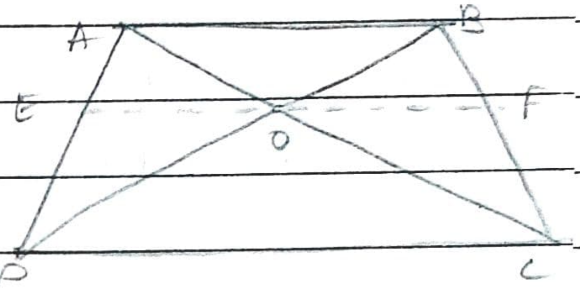
So, AB $\parallel$ EO

$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO}$ (By BPT) —— (ii)

from eq^n (i) and (ii), we get

$\frac{AO}{CO} = \frac{BO}{DO}$

$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$ (Hence, proved)



10. Given

ABCD is a quadrilateral in which its diagonal ~~enters~~ intersect each other at O ~~as~~ such that $\frac{AO}{BO} = \frac{CO}{DO}$

To prove

ABCD is a trapezium

Construction

Draw a line EF parallel to AB

Proof

$$\frac{AO}{BO} = \frac{CO}{DO} \quad \text{(Given)}$$

$$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO} \quad \text{———— (1)}$$

In $\triangle DAB$,

$EF \parallel AB$

So, $EO \parallel AB$

$\Rightarrow \dfrac{AE}{ED} = \dfrac{BO}{DO}$ (By BPT) ⟵ (II)

from eq^n (I) and (II), we get

$$\frac{AO}{CO} = \frac{AE}{ED}$$

$\therefore$ EO ∥ CD (By Converse of BPT).

But, AB ∥ EO

$\Rightarrow$ AB ∥ CD

$\therefore$ quadrilateral ABCD is a trapezium. (Proved)