COORDINATE GEOMETRY

Ex-7.1

1 i) $(2,3)$, $(4,1)$

$x_1 = 2$, $y_1 = 3$, $x_2 = 4$, $y_2 = 1$

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (1-3)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

ii) $P(-5,7)$ and $Q(-1,3)$

$x_1 = -5$, $x_2 = -1$, $y_1 = 7$, $y_2 = 3$

$PQ = \sqrt{(-1+5)^2 + (3-7)^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ units

iii) $P(a,b)$ and $Q(-a,-b)$

$x_1 = a$, $x_2 = -a$, $y_1 = b$, $y_2 = -b$

$PQ = \sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2+b^2}$ units

2. Let points be $A(0,0)$ and $B(36,15)$

$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(36-0)^2 + (15-0)^2}$

$= \sqrt{1296 + 225}$

$= \sqrt{1521} = 39$ units

3. Let the given are $A(1,5)$, $B(2,3)$ and $C(-2,-11)$.

$$AB = \sqrt{(2-1)^2+(3-5)^2}$$

$$= \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2+(-11-3)^2}$$
$$= \sqrt{(-4)^2+(-14)^2}$$
$$= \sqrt{16+196}$$
$$= \sqrt{212} = 2\sqrt{53}$$

$$AC = \sqrt{(-2-1)^2+(-11-5)^2}$$
$$= \sqrt{(-3)^2+(-16)^2}$$
$$= \sqrt{9+256}$$
$$= \sqrt{265}$$

Since $AB + BC \neq AC$

$\therefore$ Points are not collinear

4. Let $A(5,-2)$ and $B(6,4)$ and $C(7,-2)$

$$AB = \sqrt{(6-5)^2+(4+2)^2} = \sqrt{1+36} = \sqrt{37}$$
$$BC = \sqrt{(7-6)^2+(-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$
$$AC = \sqrt{(7-5)^2+(-2+2)^2} = \sqrt{4+0} = 2$$

$AB = AC$

$\therefore$ ΔABC is an isosceles triangle

5. from the figure, let the points along with coordinates be $A(-3, 4)$, $B(6, 7)$, $C(9, 4)$ and $D(6, 1)$

$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$

$DA = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{18} = 3\sqrt{2}$

diagonal $AC = \sqrt{(9-3)^2 + (4-4)^2}$
$= \sqrt{(6)^2 + (0)^2} = \sqrt{36} = 6$

diagonal $BD = \sqrt{(6-6)^2 + (1-7)^2}$
$= \sqrt{(0)^2 + (-6)^2} = \sqrt{36} = 6$

$\therefore$ $AB = BC = CD = DA = 3\sqrt{2}$

$AC = BD = 6$

$\therefore$ ABCD is a square & Champa is correct

(i) Let points be A (-1, -2), B(1, 0), C(-1, 2) and D (-3, 0)

$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AD = \sqrt{(-3+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = \sqrt{16} = 4$

$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

~~AC = BD, AB = ~~

$AC = BD$, $AB = BC = CD = AD$

$\therefore$ Quadrilateral ABCD is a square.

(ii) Lets points be A(-3, 5), B(3, 1), C(0, 3) and D(-1, -4)

$AB = \sqrt{(3+3)^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{1+49} = 5\sqrt{2}$

$AD = \sqrt{(-3+1)^2 + (5+4)^2} = \sqrt{4+81} = \sqrt{85}$

$\therefore$ Given points don't form a quadrilateral.

iii) Let points be $A(-4,5)$, $B(7,6)$, $C(4,3)$ and $D(1,2)$

$AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2+(3-6)^2} = \sqrt{9+9} = 3\sqrt{2}$

$CD = \sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$AD = \sqrt{(1-4)^2+(2-5)^2} = \sqrt{9+9} = 3\sqrt{2}$

$AC = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{4} = 2$

$BD = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = 2\sqrt{13}$

$AB = CD$, $BC = AD$ and $AC \neq BD$

$\therefore$ Quadrilateral ABCD is a parallelogram

7. Let $A(2,-5)$ and $B(-2,9)$ be given points and $P(x,0)$ such that

$$PA = PB$$

$$PA^2 = PB^2$$

$\Rightarrow (x-2)^2+(0+5)^2 = (x+2)^2+(0-9)^2$

$\Rightarrow (x-2)^2-(x+2)^2 = 81-25$

$\Rightarrow (x-2+x+2)(x-2-x-2) = 56$

$\Rightarrow (2x)(-4) = 56$

$\Rightarrow -8x = 56$

$\Rightarrow x = -7$

$\therefore P(-7,0)$

8. Points P (2, -3), Q (10, y) and PQ = 10 units

$\sqrt{(10-2)^2 + (y+3)^2} = 10$

$\Rightarrow 64 + y^2 + 9 + 6y = 100$

$\Rightarrow y^2 + 6y + 73 - 100 = 0$

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow (y-3)(y+9) = 0$

$\Rightarrow y - 3 = 0$ or $y + 9 = 0$

$\therefore y = 3$ or -9

9. Given that Q(0, 1) is equidistant from P(5, -3) and R(x, 6)

$QP = QR$

$QP^2 = QR^2$

$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

$QR = \sqrt{(x-0)^2 + (6-1)^2} = \sqrt{x^2 + 5^2} = \sqrt{(4)^2 + 5^2}$

$= \sqrt{16 + 25} = \sqrt{41}$

$PR = \sqrt{(x-5)^2 + (6+3)^2} = \sqrt{(4-5)^2 + (6+3)^2} = \sqrt{(-1)^2 + (9)^2}$

$= \sqrt{1 + 81} = \sqrt{82}$

Also,

$PR = \sqrt{(-4-5)^2+(6+3)^2} = \sqrt{(-9)^2+(9)^2} = \sqrt{162}$

$= 9\sqrt{2}$

$\therefore$ ~~$QR = \sqrt{41}$ & $PR = \sqrt{82}$,~~

$\therefore$ $QR = \sqrt{41}$ and $PR = \sqrt{82}$, $9\sqrt{2}$

10. Points A (3, 6) and B(-3, 4) are equidistant from point P(x, y).

$AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-3)^2 + (y+6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 36 + 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$\Rightarrow -6x - 6y - 12y + 8y + 45 - 25 = 0$

$\Rightarrow 12x - 4y + 20 = 0$

$\Rightarrow 3x - y + 5 = 0$