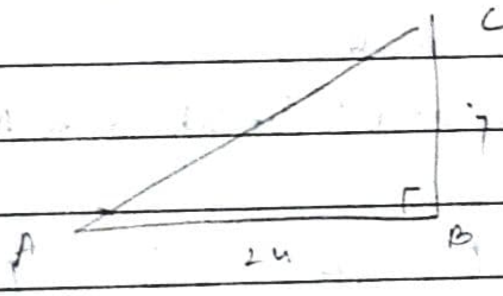
TRIGONOMETRY

Ex - 8.1

1. By Pythagoras theorem,



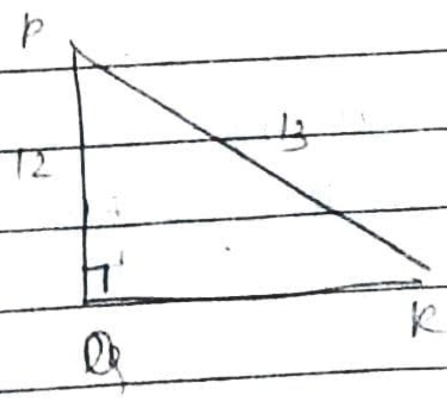
$$AC^2 = AB^2 + BC^2$$
$$= 24^2 + 7^2$$
$$= 576 + 49$$
$$= 625$$

$$AC = 25\,cm$$

i) $\sin A = \frac{7}{25}$, $\cos A = \frac{24}{25}$

ii) $\sin C = \frac{24}{25}$, $\cos C = \frac{7}{25}$

2. $PR^2 = PQ^2 + QR^2$



$13^2 = 12^2 + QR^2$

$\Rightarrow 169 - 144 = QR^2$

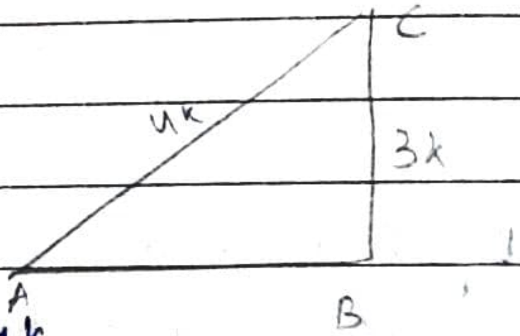
$\Rightarrow 25 = QR^2$

$\Rightarrow 5 = QR$

$\tan P = \frac{5}{12}$, $\cot R = \frac{5}{12}$

$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

3. $\sin A = \frac{3}{4} = \frac{BC}{AC}$



B B

Let BC be 3k and AC be 4k

$$\cancel{AB^2 = BC^2 + AC^2}$$
$$\cancel{= (3k)^2 + (4k)^2}$$
$$\cancel{= 9k^2 + 16k^2}$$
$$\cancel{= 25k^2}$$
$$\cancel{AB = }$$

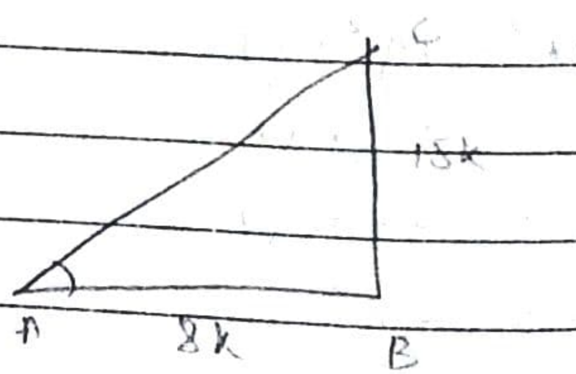
$$AB^2 = AC^2 - BC^2$$
$$= (4k)^2 - (3k)^2$$
$$= 16k^2 - 9k^2$$
$$= 7k^2$$
$$AB = \sqrt{7}k$$

$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

4. $15 \cot A = 8$

$\cot A = \frac{8}{15} = \frac{AB}{BC}$



$AB = 8k$ and $BC = 15k$

$AC^2 = AB^2 + BC^2$

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$AC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

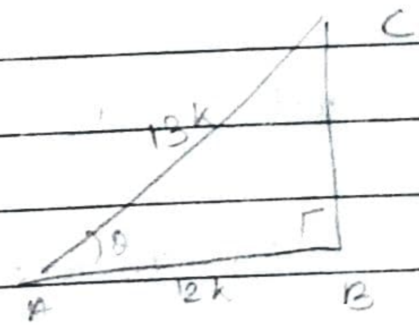
$= 289k^2$

$AC = 17k$

$\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. $\sec\theta = \frac{13}{12} = \frac{AC}{AB}$



Let $AC = 13k$ and $AB = 12k$

$AC^2 = AB^2 + BC^2$

$\cancel{= 169k^2 + 144k^2}$

$\Rightarrow 169k^2 = 144k^2 + BC^2$

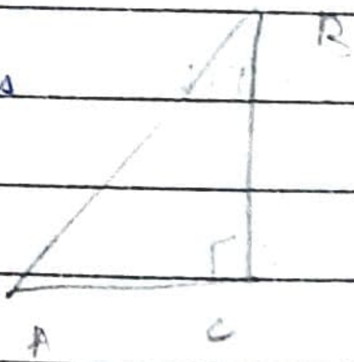
$\Rightarrow 25k^2 = BC^2$

$\Rightarrow 5k = BC$

$\sin\theta = \frac{5k}{13k} = \frac{5}{13}$, $\cos\theta = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{5k}{12k} = \frac{5}{12}$, $\operatorname{cosec}\theta = \frac{13k}{5k} = \frac{13}{5}$, $\cot\theta = \frac{12k}{5k} = \frac{12}{5}$

6. Since $\angle A$ and $\angle B$ are acute angles



$\angle C = 90°$

$\cos A = \cos B$

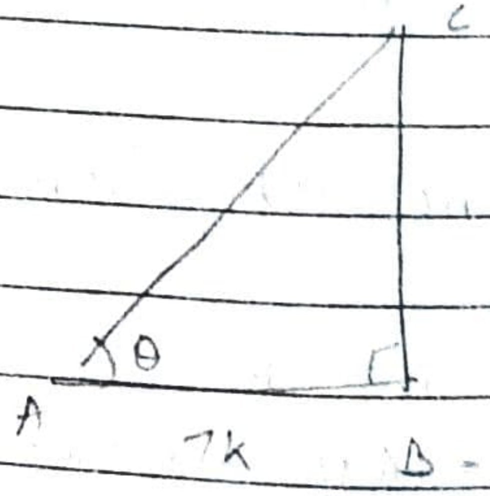
$\frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

$\Rightarrow \angle A = \angle B$ (angles opposite to equal sides are equal)

7. $\cot\theta = \frac{7}{8} = \frac{AB}{AC}$

Let $AB = 7k$ and $BC = 8k$



$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (7k)^2 + (8k)^2$

$= 49k^2 + 64k^2$

$= 113k^2$

$\Rightarrow AC = \sqrt{113}\,k$

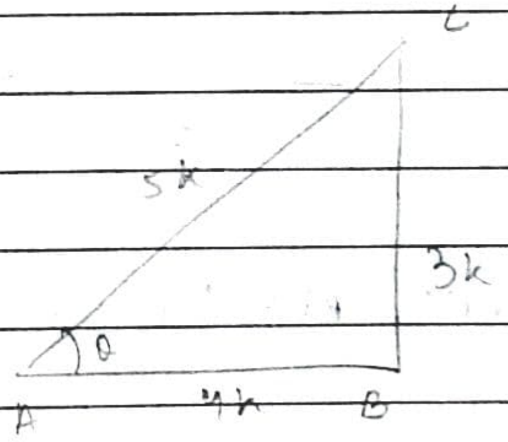
i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{49}{64}$

8. $3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3} = \frac{AB}{BC}$



Let $AB = 4k$ and $BC = 3k$

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 16k^2 + 9k^2$

$= 25k^2$

$\Rightarrow AC = 5k$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

LHS: $\frac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\left(\frac{\sin^2 A}{\cos^2 A}\right)}{1+\left(\frac{\sin^2 A}{\cos^2 A}\right)} = \frac{1-\left(\frac{\sin A}{\cos A}\right)^2}{1+\left(\frac{\sin A}{\cos A}\right)^2} = \frac{1-\left(\frac{3k}{4k}\right)^2}{1+\left(\frac{3k}{4k}\right)^2}$$

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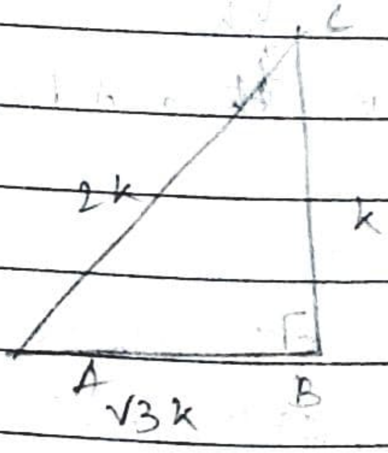
$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

RHS: $\cos^2 A - \sin^2 A$

$$= \left(\frac{4k}{5k}\right)^2 - \left(\frac{3k}{5k}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$\therefore$ LHS = RHS (Proved)



9. $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $AB = \sqrt{3}k$ and $BC = k$

$$AC^2 = AB^2 + BC^2$$
$$= 3k^2 + k^2$$

$\Rightarrow AC^2 = 4k^2$

$\Rightarrow AC = 2k$

i) $\sin A \cos C + \cos A \sin C$

$$= \left(\frac{k}{2k}\right)\left(\frac{k}{2k}\right) + \left(\frac{\sqrt{3}k}{2}\right)\left(\frac{\sqrt{3}k}{2}\right)$$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

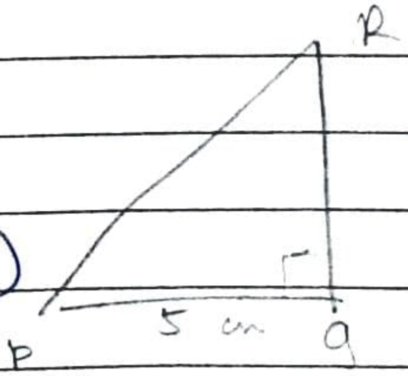
$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

ii) $\cos A \cos C - \sin A \sin C$

$= \left(\frac{\sqrt{3}k}{2}\right)\left(\frac{k}{2k}\right) - \left(\frac{k}{2k}\right)\left(\frac{\sqrt{3}k}{2}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$



10. $PR^2 = PQ^2 + QR^2$

$\Rightarrow PQ^2 = PR^2 - QR^2$

$\Rightarrow 5^2 = \cancel{RQ\ PR^2 ca} (PR - QR)(PR + QR)$

$\Rightarrow 25 = 25(PR - QR)$

$\Rightarrow 1 = PR - QR$ ——— (1)

~~So~~

$PR + QR = 25$ ——— (11) (given)

from (1) and (11),

$2PR = 26 \Rightarrow PR = 13$ cm

and $QR = 12$ cm

$\sin P = \frac{12}{13}$, $\cos P = \frac{5}{13}$, $\tan P = \frac{12}{5}$

11. i) false
ii) True
iii) false
iv) false
v) ~~falts~~ false