3. State, which of these following sets are finite and which are non-finite.

(i) $A = \{x : x \in Z \text{ and } x < 10\}$

Sol:- $A = \{x : x \in Z \text{ and } x < 10\}$

$= \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4 \ldots\ldots\}$

$= \{\ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

So, it is an infinite set.

(ii) $B = \{x : x \in W \text{ and } 5x - 3 \leq 20\}$

Sol:- $B = \{x : x \in W \text{ and } 5x - 3 \leq 20\}$

$5x - 3 \leq 20$

$\Rightarrow 5x - 3 + 3 \leq 20 + 3$ (Adding 3 to both sides)

$\Rightarrow 5x \leq 23$

$\Rightarrow \frac{5x}{5} \leq \frac{23}{5}$

$\Rightarrow x \leq \frac{23}{5}$

$\Rightarrow x \leq 4.6$

$\therefore B = \{0, 1, 2, 3, 4\}$

So, it is a finite set.

(iii) $P = \{y : y = 3x - 2, x \in N \text{ and } x > 5\}$

Sol:- $y = 3x - 2$

When $x = 6$, $y = 3 \times 6 - 2$
$= 18 - 2 = 16$

When $x = 7$, $y = 3 \times 7 - 2$
$= 21 - 2 = 19$

When $x = 8$, $y = 3 \times 8 - 2$
$= 24 - 2 = 22$

When $x = 9$, $y = 3 \times 9 - 2$
$= 27 - 2 = 25$

When $x = 10$ $y = 3 \times 10 - 2$
~~When $x = 10$~~ $= 30 - 2 = 28$

$\therefore P = \{16, 19, 22, 25, 28, \ldots\ldots\ldots\}$

So, it is a infinite set.

(iv) $M = \{r : r = \frac{3}{n} ; n \in W \text{ and } 6 < n \leq 15\}$

Sol:- $r = \frac{3}{n}$

When $n = 7$, $r = \frac{3}{7}$

When $n = 8$, $r = \frac{3}{8}$

When $n = 9$, $r = \frac{3}{9} = \frac{1}{3}$

When $n = 10$, $r = \frac{3}{10}$.

When $n = 11$, $r = \frac{3}{11}$

When $n = 12$, $r = \frac{3}{12}$

When $n = 13$, $r = \frac{3}{13}$

When $n = 14$; $r = \frac{3}{14}$

When $n = 15$; $r = \frac{3}{15}$

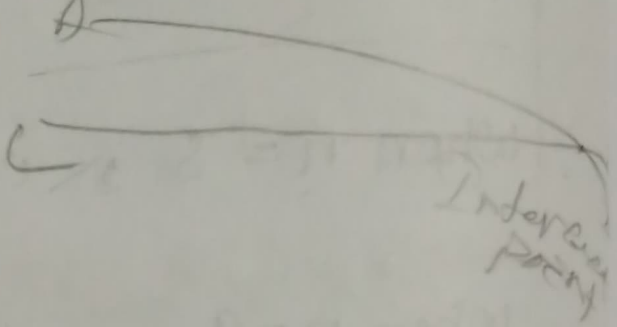
$\therefore M = \{\frac{3}{7}, \frac{3}{8}, \frac{3}{9}, \frac{3}{10}, \frac{3}{11}, \frac{3}{12}, \frac{3}{13}, \frac{3}{14}, \frac{3}{15}\}$

So, it is a finite set.

4. Find, which of the following sets are singleton sets:

(i) The set of points of interaction of two non-parallel st. lines in the same plane.

Ans. Single-ton set



(ii) $A = \{x : 7x - 3 = 11\}$

Ans- $7x - 3 = 11$

$\Rightarrow 7x = 11 + 3$

$\Rightarrow 7x = 14$

$\Rightarrow x = \frac{14}{7}$

$\Rightarrow x = 2$

So, $A = \{2\}$

Hence it is an singleton set.

(iii) $B = \{y : 2y + 1 < 3 \text{ and } y \in W\}$

Ans- $2y + 1 < 3$

$\Rightarrow 2y + 1 - 1 < 3 - 1$

$\Rightarrow 2y < 2$

$\Rightarrow y < \frac{2}{2}$

$\Rightarrow y < 1$

$\therefore B = \{0\}$

Hence it is a singleton set.

5. Find, which of the following sets are empty:

(i) The set of points of intersection of two parallel lines.

Ans- It is an empty set because two parallel lines never intersect each other anywhere.

(ii) $A = \{x : x \in N \text{ and } 5 < x \leq 6\}$

Ans- Since $5 < x \leq 6$

So $x = 6$

$\therefore A = \{6\}$ Hence, it is not an empty set.

(iii) $B = \{x : x^2 + 4 = 0, x \in N\}$

Ans. $x^2 + 4 = 0$

$\Rightarrow x^2 + 4 - 4 = 0 - 4$

$\Rightarrow x^2 = -4$

$\Rightarrow x = \sqrt{-4}$ which is not a natural number

So, $B = \{\}$

Hence it is an empty set.

(iv) $C = \{$ Even numbers between 6 and 10 $\}$

Ans. $C = \{8\}$

Hence, it is not an empty set.

(v) $D = \{$ prime numbers between 7 and 11. $\}$

Ans. There is no prime number between 7 and 11.

So, $D = \{\}$

Hence, it is an empty set.

6.(i) Are the Set $A=\{4,5,6\}$ and $B=\{x: x^2-5x-6=0\}$ disjoint?

Ans- Set $A=\{4,5,6\}$ Given

$B=\{x: x^2-5x-6=0\}$

Since, $x^2-5x-6=0$

$\Rightarrow x^2-6x+x-6=0$

$\Rightarrow x(x-6)+1(x-6)=0$

$\Rightarrow (x-6)(x+1)=0$

Either, $x-6=0$ or $x+1=0$

$\Rightarrow x=6$ $\Rightarrow x=-1$

$\therefore B=\{6,-1\}$

In the set A and B, one element i.e. 6 is common. Hence, set A and B are not disjoint set.

(ii) Are the sets $A = \{b, c, d, e\}$ and $B = \{x : x \text{ is a letter in the word 'MASTER'}\}$ joint?

Ans Set $A = \{b, c, d, e\}$ Given

$B = \{x : x \text{ is a letter, in the word 'MASTER'}\}$

$\therefore B = \{m, a, s, t, e, r\}$

In the above set A and B one element that is e is common. So, it is a joint set.

7. State whether the following pairs of sets A are equivalent or not:

(i) $A = \{x : x \in N \text{ and } 11 \geq 2x - 1\}$ and $B = \{y : y \in W \text{ and } 3 \leq y \leq 9\}$

Ans $A = \{x : x \in N \text{ and } 11 \geq 2x - 1\}$

Since, $11 \geq 2x - 1$

$\Rightarrow 11 + 1 \geq 2x - 1 + 1$

$\Rightarrow 12 \geq 2x$

$\Rightarrow \frac{12}{2} \geq x$

$\Rightarrow 6 \geq x$

$\Rightarrow x \leq 6$

$\therefore A = \{1, 2, 3, 4, 5, 6\}$ $\boxed{n(A) = 6}$

$B = \{y : y \in W \text{ and } 3 \leq y \leq 9\}$

$\because 3 \leq y \leq 9$

$B = \{3, 4, 5, 6, 7, 8, 9\}$ $\boxed{n(B) = 7}$

So, $n(A) \neq n(B)$

Hence, set A and B are not equivalent.

(ii) Set of integer and set of natural numbers

Ans Set of Z $= \{-1, -2, -3, -4, -5 \ldots\ldots\ldots\}$ n = Infinite

Set of N $= \{1, 2, 3, 4, 5, 6 \ldots\ldots\ldots\ldots\}$ n = Infinite

So, $n(Z) = n(N)$

Hence, set of integers and set of natural numbers are equivalent.

(iii) Set of whole numbers and set of multiple of 3.

Ans- Set of $W = \{0, 1, 2, 3, 4, 5 \ldots\ldots\ldots\}$ $n = \text{Infinite}$

Set of multiple of 3

$= \{3, 6, 9, 12, 15, 18, 21, \ldots\ldots\ldots\}$ $n = \text{Infinite}$

Set of whole numbers and multiple of 3 are equivalent because both sets have infinite number of element.

(iv) $P = \{5, 6, 7, 8\}$ and $M = \{x : x \in W \text{ and } x \leq 4\}$

Ans- $P = \{5, 6, 7, 8\}$

$n(P) = 4$

$M = \{x : x \in W \text{ and } x \leq 4\}$

Since $x \leq 4$, so $x = 0, 1, 2, 3, 4$

$\therefore M = \{0, 1, 2, 3, 4\}$

$n(M) = 5$

Here $n(P) \neq n(M)$

So, Set P and M is not equivalent.

8. State whether the following pairs of sets are equal or not:

(i) $A = \{2, 4, 6, 8\}$ and $B = \{2n : n \in N \text{ and } n < 5\}$

Ans- When $n = 1$, $2n = 2 \times 1 = 2$

When $n = 2$, $2n = 2 \times 2 = 4$

When $n = 3$, $2n = 2 \times 3 = 6$

When $n = 4$, $2n = 2 \times 4 = 8$

$\therefore B = \{2, 4, 6, 8\}$

Now we see that elements of sets A and B are the same

$\therefore$ Sets A and B are equal.

(ii) $M = \{x : x \in W \text{ and } x + 3 < 8\}$ and $N = \{y : y = 2n - 1, n \in N \text{ and } n < 5\}$

Ans- $x + 3 < 8$

$\Rightarrow x + 3 - 3 < 8 - 3$

$\Rightarrow x < 5$

$M = \{0, 1, 2, 3, 4\}$

$N = \{y : y = 2n - 1, n \in N \text{ and } n < 5\}$

$y = 2n - 1$

When $n = 1$ $\quad y = 2 \times 1 - 1$

$\Rightarrow 2 - 1 = 1$

When $n = 2$ $\quad y = 2 \times 2 - 1$

$\Rightarrow 4 - 1 = 3$

When $n=3$, $y = 2\times 3 - 1$

$\Rightarrow y = 6 - 1 = 5$

When $n=4$, $y = 2\times 4 - 1$

$\Rightarrow y = 8 - 1 = 7$

$\therefore N = \{1, 3, 5, 7\}$

Now, we see that elements of sets M and N are not the same.

$\therefore$ Sets M and N are not equal

(iii) $E = \{x : x^2 + 8x - 9 = 0\}$ and $F = \{1, -9\}$

Ans

$x^2 + 8x - 9 = 0$

$\Rightarrow x^2 + 9x - x - 9 = 0$

$\Rightarrow x(x+9) - 1(x+9) = 0$

$\Rightarrow (x-1)(x+9) = 0$

$x - 1 = 0$ or $x + 9 = 0$

$\Rightarrow x = 1$ $\Rightarrow x = -9$

$\therefore E = \{1, -9\}$

$F = \{1, -9\}$

So, we see that the elements of E and F are same.

$\therefore$ Sets E and F are equal.

(iv) $A = \{x : x \in N, x < 3\}$ and $B = \{y : y^2 - 3y + 2 = 0\}$

Ans-

$y^2 - 3y + 2 = 0$

$\Rightarrow y^2 - 2y - y + 2 = 0$

$\Rightarrow y(y-2) - 1(y-2) = 0$

$\Rightarrow (y-2)(y-1) = 0$

$\therefore y - 2 = 0$ or $y - 1 = 0$

$\Rightarrow y = 2$ $\Rightarrow y = 1$

$B = \{1, 2\}$

9. State whether each of the following sets is a finite or an infinite set:

(i) The set of multiples of 8.

Ans

$= \{8, 16, 24, 32, \ldots\ldots\}$

So, it is an infinite set.

(ii) The set of integers less than 10.

Ans-

$= \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, \ldots\ldots\ldots\ldots\}$

It is a infinite set.

(iii) The set of whole numbers less than 12.

Ans. $= \{11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$

So, it is a finite set.

(iv) $\{x : x = 3n - 2, n \in N, n \leq 8\}$

Ans-

When $n = 0$, $x = 3 \times 0 - 2$
$\Rightarrow 0 - 1 = -2$

When $n = 1$, $x = 3 \times 1 - 2$
$\Rightarrow 3 - 2 = 1$

When $n = 2$, $x = 3 \times 2 - 2$
$\Rightarrow 6 - 2 = 4$

When $n = 3$, $x = 3 \times 3 - 2$
$\Rightarrow 9 - 2 = 7$

When $n = 4$, $x = 3 \times 4 - 2$
$\Rightarrow 12 - 2 = 10$

When $n = 5$, $x = 3 \times 5 - 2$
$= 15 - 2 = 13$

When $n = 6$, $x = 3 \times 6 - 2$
$\Rightarrow 18 - 2 = 16$

When $n = 7$, $x = 3 \times 7 - 2$
$\Rightarrow 21 - 2 = 19$

When $n = 8$, $x = 3 \times 8 - 2$
$\Rightarrow 24 - 2 = 22$

$= \{-2, 1, 4, 7, 10, 13, 16, 19, 22\}$

So, it is a finite set.

(v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

Ans-

When $n = 8$, $x = 3 \times 8 - 2$
$\Rightarrow 24 - 2 = 22$

When $n = 7$, $x = 3 \times 7 - 2$
$\Rightarrow 21 - 2 = 19$

When $n = 6$, $x = 3 \times 6 - 2$
$\Rightarrow 18 - 2 = 16$

When $n = 5$, $x = 3 \times 5 - 2$
$\Rightarrow 15 - 2 = 13$

When $n = 4$, $x = 3 \times 4 - 2$
$\Rightarrow 12 - 2 = 10$

When $n = 3$, $x = 3 \times 3 - 2$
$\Rightarrow 9 - 2 = 6$

When $n = 2$, $x = 3 \times 2 - 2$
$\Rightarrow 6 - 2 = 4$

When $n = 1$, $x = 3 \times 1 - 2$
$\Rightarrow 3 - 2 = 1$

When $n = 0$, $x = 3 \times 0 - 2$
$\Rightarrow 0 - 2 = -2$

When $n = -1$, $x = 3 \times -1 - 2$
$\Rightarrow -3 - 2 = -5$

$= \{ 22, 19, 16, 13, 10, 7, 4, 1, -2, -5, \ldots\ldots\ldots \}$

So, it is a infinite set.

(vi) $\{x : x = \frac{n-2}{n+1}, n \in W\}$

Ans- When, $n = 0$, $x = \frac{0-2}{0+1} = \frac{-2}{1} = -2$

When, $n = 1$, $x = \frac{1-2}{1+1} = \frac{-1}{2}$

When, $n = 2$, $x = \frac{2-2}{2+1} = \frac{0}{3} = 0$

When, $n = 3$, $x = \frac{3-2}{3+1} = \frac{1}{4}$

When, $n = 4$, $x = \frac{4-2}{4+1} = \frac{2}{5}$

$= \{ -2, 0, \frac{1}{4}, \frac{2}{5} \ldots\ldots\ldots\ldots \}$

So, it is a infinite set.

10. Answer, whether the following statements are true or false. Give reasons.

(ii) If E = {factors of 16} and F = {factors of 20}, than, E = F.

Ans. E = {Factors of 16}
= {1, 2, 4, 8, 16}

1 × 16 = 16
2 × 8 = 16
4 × 4 = 16

F = {Factors of 20}
= {1, 2, 4, 5, 10, 20}

1 × 20 = 20
2 × 10 = 20
4 × 5 = 20

Now we can see that the elements of set E and set F are not equal.

"If E = {Factors of 16} = F = {Factors of 20}"

So, statement is false.

(i) The set of even natural numbers less than 21 and set of odd natural numbers less than 21 are equivalent sets.

Ans. Set of even natural number less than 21
= {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

Cardinal no of set = 10

Set of odd natural numbers less than 21.

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

Cardinal No. of set $= 10$

Now, we see that cardinal numbers of both these sets $= 10$

So, it is a true statement.

(vii) The set $A = \{$ integers less than $20\}$ is a finite set.

Ans- $A = \{$ integers less than $20\}$

$= \{19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, \ldots\ldots\ldots\ldots\}$

So, it is a false statement.

(iv) If $A = \{x : x$ is an even prime number$\}$, then set A is empty.

Ans. $A = \{x : x$ is an even prime number$\} = \{2\}$

So, it is a false statement.

(v) The set of odd prime is the empty set.

Ans- Set of odd prime numbers

$= \{3, 5, 7, 11, 13, 17, 19, 23, \ldots\ldots\ldots\}$

So, it is a false statement.

(vi) The set of squares of integers and the set of whole numbers are equal sets.

Ans- Integer | Square of integers | whole no.

Integers

$0 : (0)^2 = 0$ — 0

$\pm 1 : (\pm 1)^2 = 1$ — 1

$\pm 2 : (\pm 2)^2 = 4$ — 2

$\pm 3 : (\pm 3)^2 = 9$ — 3

$\pm 4 : (\pm 4)^2 = 16$ — 4

$\pm 5 : (\pm 5)^2 = 25$ — 5

..... : ....

..... : ....

Set of squares of integers

$= \{0, 1, 4, 9, 16, 25, \ldots\ldots\}$

Set of whole numbers $= \{0, 1, 2, 3, 4, 5, \ldots\ldots\ldots\}$

So, it is a false statement.

(vii) If $n(P) = n(M)$, then $P \longrightarrow M$.

Ans. It means no of elements of set P = No of elements of set M.

Sets P and M are equivalent.

So, it is a true statement.

(viii) If set P = set M, then $n(P) = n(M)$

Ans. It means sets P and M are equal. Equal sets are equivalent also.

No of elements of set P = No of elements of set M.

So, it is a true statement.

(ix) $n(A) = n(B) \Rightarrow A = B$.

Ans. Number of elements of set A

= Number of elements of set B

Given sets are equivalent but not equal.

So, it is a false statement.