Ex-6(D)

1. Given $A=\{x : x \in N \text{ and } 3 < x \leq 6\}$ and $B=\{x : x \in W \text{ and } x < 4\}$: Find

(i) Sets A and B in roaster form

Ans. $A = \{4,5,6\}$

$B = \{0,1,2,3\}$

(ii) $A \cup B = \{0,1,2,3,4,5,6\}$

(iii) $A \cap B = (\phi)$

(iv) $A - B = (4,5,6)$

(v) $B - A = (0,1,2,3)$

2. If $P=\{x : x \in W \text{ and } 4 \leq x \leq 8\}$ and $Q=\{x : x \in N \text{ and } x < 6\}$. Find:

(i) $P \cup Q$ and $P \cap Q$

Ans- $P = \{4,5,6,7,8\}$

$Q = \{1,2,3,4,5\}$

$P \cup Q = (1,2,3,4,5,6,7,8)$

$P \cap Q = (4,5)$

(ii) Is $(P \cup Q) \supset (P \cap Q)$

Ans Yes, all the element of set $P \cup Q$ are contained in the set $P \cap Q$. Therefore $P \cup Q$ is a proper subset of $P \cap Q$.

3. If $A = \{5, 6, 7, 8, 9\}$, $B = \{x : 3 < x < 8 \text{ and } x \in W\}$ and $C = \{x : x \leq 5 \text{ and } x \in N\}$.

Find:

$A = \{5, 6, 7, 8, 9\}$

$B = \{4, 5, 6, 7\}$

$C = \{1, 2, 3, 4, 5\}$

(i) $A \cup B$ and $(A \cup B) \cup C$

Ans $A \cup B = (4, 5, 6, 7, 8, 9)$

$(A \cup B) \cup C = (1, 2, 3, 4, 5, 6, 7, 8, 9)$

(ii) $B \cup C$ and $A \cup (B \cup C)$

Ans $B \cup C = (1, 2, 3, 4, 5, 6, 7, 8, 9)$

$A \cup (B \cup C) = (1, 2, 3, 4, 5, 6, 7, 8, 9)$

(iii) $A \cap B$ and $(A \cap B) \cap C$

Ans- $A \cap B = (5, 6, 7)$

$(A \cap B) \cap C = (5)$

(iv) $B \cap C$ and $A \cap (B \cap C)$

Ans- $B \cap C = (4, 5)$

$A \cap (B \cap C) = (5)$

(v) Is $(A \cup B) \cup C = A \cup (B \cup C)$?

Ans- $(A \cup B) \cup C = (1, 2, 3, 4, 5, 6, 7, 8, 9)$

$A \cup (B \cup C) = (1, 2, 3, 4, 5, 6, 7, 8, 9)$

Yes, these are equal.

(vi) Is $(A \cap B) \cap C = A \cap (B \cap C)$?

Ans- $(A \cap B) \cap C = A \cap (B \cap C)$

$\{5\} = \{5\}$

$A \cap B = \{5, 6, 7\}$

$LHS = (A \cap B) \cap C = \{5\}$

$B \cap C = \{4, 5\}$

RHS = $A \cap (B \cap C) = \{5\}$

LHS = RHS

Yes, these are equal.

4. Given, $A = \{0, 1, 2, 4, 5\}$,

$B = \{0, 2, 4, 6, 8\}$

$C = \{0, 3, 6, 9\}$

(a) $A \cup (B \cup C) = (A \cup B) \cup C$ i.e, the union of sets is associative.

Ans

$B \cup C = \{0, 2, 4, 6, 8\} \cup \{0, 3, 6, 9\}$

$= \{0, 2, 3, 4, 6, 8, 9\}$

LHS $\rightarrow A \cup (B \cup C) = \{0, 1, 2, 4, 5\} \cup \{0, 2, 3, 4, 6, 8, 9\}$

$= \{0, 1, 2, 3, 4, 5, 6, 8, 9\} \longrightarrow$ eqn (1)

$A \cup B = \{0, 1, 2, 4, 5\} \cup \{0, 2, 4, 6, 8\}$

$= \{0, 1, 2, 4, 5, 6, 8\}$

RHS $\rightarrow (A \cup B) \cup C = \{0, 1, 2, 4, 5, 6, 8\} \cup \{0, 3, 6, 9\}$

$= \{0, 1, 2, 3, 4, 5, 6, 8, 9\} \longrightarrow$ eqn (2)

Here, LHS = RHS. So, the union of sets is associative.

$A \cup (B \cup C) = (A \cup B) \cup C$

(iii) $A \cap (B \cap C) = (A \cap B) \cap C$ i.e. the intersection of sets is associative.

Ans

$B \cap C = \{0, 2, 4, 6, 8\} \cap \{0, 3, 6, 9\}$

$= \{0, 6\}$

$LHS \rightarrow A \cap (B \cap C) = \{0, 1, 2, 4, 5\} \cap \{0, 6\}$

$= \{0\} \longrightarrow$ eqn (1)

$A \cap B = \{0, 1, 2, 4, 5\} \cap \{0, 2, 4, 6, 8\}$

$= \{0, 2, 4\}$

$RHS \longrightarrow (A \cap B) \cap C = \{0, 2, 4\} \cap \{0, 3, 6, 9\}$

$= \{0\} \longrightarrow$ eqn (2)

From, equation 1 and 2,

Here, LHS = RHS. So, the intersection of sets is associative.

5. If $A=\{x \in W : 5 < x < 10\}$, $B=\{3,4,5,6,7\}$ and $C=\{x=2n : n \in N \text{ and } n \leq 4\}$. Find.

(i) ~~$A \cap (B \cup C)$~~

Ans ~~$B \cup C$~~

$A=\{x \in W : 5 < x < 10\}$

$=\{6,7,8,9\}$

$B=\{3,4,5,6,7\}$

$C=\{x=2n ; n \in N \text{ and } n \leq 4\}$

$x=2n$

when, $n=1$, $x=2\times1=2$

$n=2$, $x=2\times2=4$

$n=3$, $x=2\times3=6$

$n=4$, $x=2\times4=8$

$\therefore C=\{2,4,6,8\}$

(i) $B \cup C = \{3,4,5,6,7\} \cup \{2,4,6,8\}$

$=\{2,3,4,5,6,7,8\}$

$A \cap (B \cup C) = \{6,7,8,9\} \cap \{2,3,4,5,6,7,8\}$

$\Rightarrow A \cap (B \cap C) = \{6,7,8\}$

(ii) $(B \cup A) \cap (B \cup C)$

$B \cup A = \{3,4,5,6,7\} \cup \{6,7,8,9\}$

$= \{3,4,5,6,7,8,9\}$

$(B \cup A) \cap (B \cup C)$

$= \{3,4,5,6,7,8,9\} \cap \{2,3,4,5,6,7,8\}$

$= \{3,4,5,6,7,8\}$

(iii) $B \cup (A \cap C)$

$(A \cap C) = \{6,7,8,9\} \cap \{2,4,6,8\}$

$= \{6,8\}$

$B \cup (A \cap C) = \{3,4,5,6,7\} \cup \{6,8\}$

$= \{3,4,5,6,7,8\}$

(iv) $(A \cap B) \cup (A \cap C)$

$A \cap B = \{6,7,8,9\} \cap \{3,4,5,6,7\}$

$= \{6,7\}$

$(A \cap B) \cup (A \cap C)$

$= \{6,7\} \cup \{6,8\}$

$= \{6,7,8\}$

6. If, P = {factors of 36} and Q = {factors of 48}:

P = {factors of 36}
= {1, 2, 3, 4, 6, 9, 12, 18, 36}

Q = {factors of 48}
= {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

(i) P ∪ Q =
= {1, 2, 3, 4, 6, 9, 12, 18, 36} ∪ {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

= {1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48}

(ii) P ∩ Q
= {1, 2, 3, 4, 6, 9, 12, 18, 36} ∩ {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
= {1, 2, 3, 4, 6, 12}

(iii) Q - P
= {1, 2, 3, 4, 6, 8, 12, 16, 24, 48} - {1, 2, 3, 4, 6, 9, 12, 18, 36}
= {8, 16, 24, 48}

(iv) P' ∩ Q
= Q - P
= {1, 2, 3, 4, 6, 8, 12, 16, 24, 48} - {1, 2, 3, 4, 6, 9, 12, 18, 36}
= {8, 16, 24, 48}

7. If $A=\{6,7,8,9\}$, $B=\{4,6,8,10\}$ and $C=\{x: x \in N: 2<x \leq 7\}$; Find.

$A = \{6,7,8,9\}$

$B = \{4,6,8,10\}$

$C = \{x: x \in N: 2<x \leq 7\}$

$= \{3,4,5,6,7\}$

(i) $A-B$

Ans $A-B = \{6,7,8,9\} - \{4,6,8,10\}$

$= \{7,9\}$

(ii) $B-C$

Ans $B-C = \{4,6,8,10\} - \{3,4,5,6,7\}$

$= \{8,10\}$

(iii) $B-(A-C)$

Ans $A-C = \{6,7,8,9\} - \{3,4,5,6,7\}$

$= \{8,9\}$

$B-(A-C) = \{4,6,8,10\} - \{8,9\}$

$= \{4,6,10\}$

(iv) $A-(B \cup C)$

Ans

$B \cup C = \{4,6,8,10\} \cup \{3,4,5,6,7\}$

$= \{3,4,5,6,7,8,10\}$

$A-(B \cup C) = \{6,7,8,9\} - \{3,4,5,6,7,8,10\}$

$= \{9\}$

(v) $B-(A \cap C)$

Ans

$A \cap C = \{6,7,8,9\} \cap \{3,4,5,6,7\}$

$= \{6,7\}$

$B-(A \cap C) = \{4,6,8,10\} - \{6,7\}$

$= \{4,8,10\}$

(vi) $B-B$

Ans

$B-B = \{4,6,8,10\} - \{4,6,8,10\}$

$= \phi$

8. If $A=\{1,2,3,4,5\}$

$B=\{2,4,6,8\}$

$C=\{3,4,5,6\}$

Verify:

(i) $A-(B\cup C)=(A-B)\cap(A-C)$

Ans $B\cup C=\{2,4,6,8\}\cup\{3,4,5,6\}$

$=\{2,3,4,5,6,8\}$

$A-(B\cup C)=\{1,2,3,4,5\}-\{2,3,4,5,6,8\}$

$=\{1\}$

$A-B=\{1,3,3,4,5\}-\{2,4,6,8\}$

$=\{1,3,5\}$

$A-C=\{1,2,3,4,5\}-\{3,4,5,6\}$

$=\{1,2\}$

$(A-B)\cap(A-C)=\{1,3,5\}-\{1,2\}=\{1\}$

$A-(B\cup C)=(A-B)\cap(A-C)$

(iii) Ans-

(a) $A-(B\cap C)=(A-B)\cup(A-C)$

$B\cap C=\{2,4,6,8\}\cap\{3,4,5,6\}$

$=\{4,6\}$

$A-(B\cap C)=\{1,2,3,4,5\}-\{4,6\}$

$=\{1,2,3,5\}$

$A-B=\{1,2,3,4,5\}-\{2,4,6,8\}$

$=\{1,3,5\}$

$A-C=\{1,2,3,4,5\}-\{3,4,5,6\}$

$=\{1,2\}$

$(A-B)\cup(A-C)=\{1,3,5\}\cup\{1,2\}$

$=\{1,2,3,5\}$

$A-(B\cap C)=(A-B)\cup(A-C)$

Q. Given $A=\{x \in N : x < 6\}$, $B=\{3,6,9\}$ and $C=\{x \in N : 2x-5 \leq 8\}$. Show that:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans-

$A=\{x \in N : x < 6\}$

$=\{1,2,3,4,5\}$

$B=\{3,6,9\}$

$C=\{x \in N : 2x-5 \leq 8\}$

$\Rightarrow 2x-5 \leq 8$

$\Rightarrow 2x \leq 8+5$

$\Rightarrow 2x \leq 13$

$\Rightarrow x \leq \frac{13}{2}$

$\Rightarrow x \leq 6.5$

$\therefore C=\{1,2,3,4,5,6\}$

$B \cap C = \{3,6,9\} \cap \{1,2,3,4,5,6\}$

$=\{3,6\}$

$A \cup (B \cap C) = \{1,2,3,4,5\} \cup \{3,6\}$
$= \{1,2,3,4,5,6\}$

$A \cup B = \{1,2,3,4,5\} \cup \{3,6,9\}$
$= \{1,2,3,4,5,6,9\}$

$A \cup C = \{1,2,3,4,5\} \cup \{1,2,3,4,5,6\}$
$= \{1,2,3,4,5,6\}$

$\therefore (A \cup B) \cap (A \cup C)$

$= \{1,2,3,4,5,6,9\} \cap \{1,2,3,4,5,6\}$
$= \{1,2,3,4,5,6\}$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$B \cup C = \{3,6,9\} \cup \{1,2,3,4,5,6\}$
$= \{1,2,3,4,5,6,9\}$

$A \cap (B \cup C) = \{1,2,3,4,5\} \cap \{1,2,3,4,5,6,9\}$
$= \{1,2,3,4,5\}$

Now $A \cap C = \{1,2,3,4,5\} \cap \{1,2,3,4,5,6\}$
$= \{1,2,3,4,5\}$

$A \cap B = \{1,2,3,4,5\} \cap \{3,6,9\}$
$= \{3\}$

$$(A \cap B) \cup (A \cap C) = \{3\} \cup \{1,2,3,4,5\}$$
$$= \{1,2,3,4,5\}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$