

4. Expand:

$$(1) (2a + b)^3$$

$$= (2a + b)(2a + b)^2$$

$$= (2a + b) \{ (2a)^2 + (b)^2 + (2 \times 2a \times b) \}$$

$$= (2a + b)(4a^2 + b^2 + 4ab)$$

$$= 2a(4a^2 + b^2 + 4ab) + b(4a^2 + b^2 + 4ab)$$

$$= 8a^3 + 2ab^2 + 8a^2b + 4a^2b + b^3 + 4ab^2$$

$$= 8a^3 + (2ab^2 + 4ab^2) + (8a^2b + 4a^2b) + b^3$$

$$= 8a^3 + 6ab^2 + 12a^2b + b^3$$

$$(ii) (a-2b)^3$$

$$= (a-2b)(a-2b)^2$$

$$= (a-2b)\{(a^2 + (-2b)^2 - (2 \times a \times 2b))\}$$

$$= (a-2b)(a^2 + 4b^2 - 4ab)$$

$$= a(a^2 + 4b^2 - 4ab) - 2b(a^2 + 4b^2 - 4ab)$$

$$= a^3 + 4ab^2 - 4a^2b - 2a^2b - 8b^3 + 8ab^2$$

$$= a^3 + (4ab^2 + 8ab^2) - (4a^2b + 2a^2b) - 8b^3$$

$$= a^3 + 12ab^2 - 6a^2b - 8b^3$$

$$\begin{aligned} \text{(iii)} \quad & (3x-2y)^3 \\ &= (3x-2y)(3x-2y)^2 \\ &= (3x-2y)(9x^2+4y^2-12xy) \\ &= 3x(9x^2+4y^2-12xy) - 2y(9x^2+4y^2-12xy) \\ &= 27x^3+12xy^2-36x^2y-18x^2y-8y^3+24xy^2 \\ &= 27x^3+12xy^2+24xy^2-36x^2y-18x^2y-8y^3 \\ &= 27x^3+36xy^2-54x^2y-8y^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & (x+5y)^3 \\ &= (x+5y)(x+5y)^2 \\ &= (x+5y)(x^2+25y^2+10xy) \\ &= x(x^2+25y^2+10xy) + 5y(x^2+25y^2+10xy) \\ &= x^3+25xy^2+10x^2y+5x^2y+125y^3+50xy^2 \\ &= x^3+25xy^2+50xy^2+10x^2y+5x^2y+125y^3 \\ &= x^3+75xy^2+15x^2y+125y^3 \end{aligned}$$

classmate  
Date \_\_\_\_\_  
Page \_\_\_\_\_

$$(v) \left(a + \frac{1}{a}\right)^3$$

$$= \left(a + \frac{1}{a}\right) \left(a + \frac{1}{a}\right)^2$$

$$= \left(a + \frac{1}{a}\right) \left(a^2 + \frac{1}{a^2} + 2\right)$$

$$= a \left(a^2 + \frac{1}{a^2} + 2\right) + \frac{1}{a} \left(a^2 + \frac{1}{a^2} + 2\right)$$

$$= a^3 + \frac{a^1}{a^2} + 2a + \frac{a^2 a}{a} + \frac{1}{a^3} + \frac{2}{a}$$

$$= a^3 + \frac{1}{a} + 2a + a + \frac{1}{a^3} + \frac{2}{a}$$

$$= a^3 + \frac{1}{a} + 3a + \frac{1}{a^3} + \frac{2}{a}$$

$$= a^3 + \frac{1}{a} + \frac{2}{a} + 3a + \frac{1}{a^3}$$

$$= a^3 + \frac{3}{a} + 3a + \frac{1}{a^3}$$

$$(vi) \left(2a - \frac{1}{2a}\right)^3$$

$$= \left(2a - \frac{1}{2a}\right) \left(2a - \frac{1}{2a}\right)^2$$

$$= \left(2a - \frac{1}{2a}\right) \left(4a^2 + \frac{1}{4a^2} - 2\right)$$

$$= 2a \left(4a^2 + \frac{1}{4a^2} - 2\right) - \frac{1}{2a} \left(4a^2 + \frac{1}{4a^2} - 2\right)$$

$$= 8a^3 + \frac{2a}{4a^2} - 4a - \frac{4a^2}{2a} - \frac{1}{8a^3} + \frac{2}{2a}$$

$$= 8a^3 + \frac{1}{2a} - 4a - 2a - \frac{1}{8a^3} + \frac{1}{a}$$

$$= 8a^3 + \frac{1}{2a} + \frac{1}{a} - 6a - \frac{1}{8a^3}$$

$$= 8a^3 + \frac{3}{2a} - 6a - \frac{1}{8a^3}$$

5. Find the cube of:

(i)  $a+2$  (Formula for  $(a+b)^3: a^3 + 3a^2b + 3ab^2 + b^3$ )

$$(a+2)^3$$

$$= (a)^3 + (2)^3 + 3 \times (a)^2 \times 2 + 3 \times a \times (2)^2$$

$$= a^3 + 8 + 6a^2 + 12a$$

(ii)  $(2a - \frac{1}{2})$  (Formula for  $(a-b)^3: a^3 - b^3 - 3a^2b + 3ab^2$ )

$$(2a-1)^3$$

$$= (2a)^3 - (1)^3 - 3 \times (2a)^2 \times 1 + 3 \times 2a \times (1)^2$$

$$= 8a^3 - 1 - 12a^2 + 6a$$

(iii)  $2a+3b$

$$(2a+3b)^3$$

$$= (2a)^3 + (3b)^3 + 3 \times (2a)^2 \times 3b + 3 \times 2a \times (3b)^2$$

$$= 8a^3 + 27b^3 + 36a^2b + 54ab^2$$

$$(ix) (3b - 2a)$$

$$= (3b)^3 - (2a)^3 - 3 \times (3b)^2 \times 2a + 3 \times 3b \times (2a)^2$$

$$= 27b^3 - 8a^3 - 54b^2a + 36a^2b$$

$$(v) 2x + \frac{1}{x}$$

$$= \left(2x + \frac{1}{x}\right)^3$$

$$= (2x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times (2x)^2 \times \frac{1}{x} + 3 \times 2x \times \left(\frac{1}{x}\right)^2$$

$$= 8x^3 + \frac{1}{x^3} + 3 \times 4x^2 \times \frac{1}{x} + 3 \times 2x \times \frac{1}{x^2}$$

$$= 8x^3 + \frac{1}{x^3} + 12x + \frac{6}{x}$$

$$(vi) x - \frac{1}{2}$$

$$\left(x - \frac{1}{2}\right)^3$$

$$= (x)^3 - \left(\frac{1}{2}\right)^3 - 3 \times (x)^2 \times \frac{1}{2} + 3 \times x \times \left(\frac{1}{2}\right)^2$$

$$= x^3 - \frac{1}{8} - \frac{3x^2}{2} + \frac{3x}{4}$$