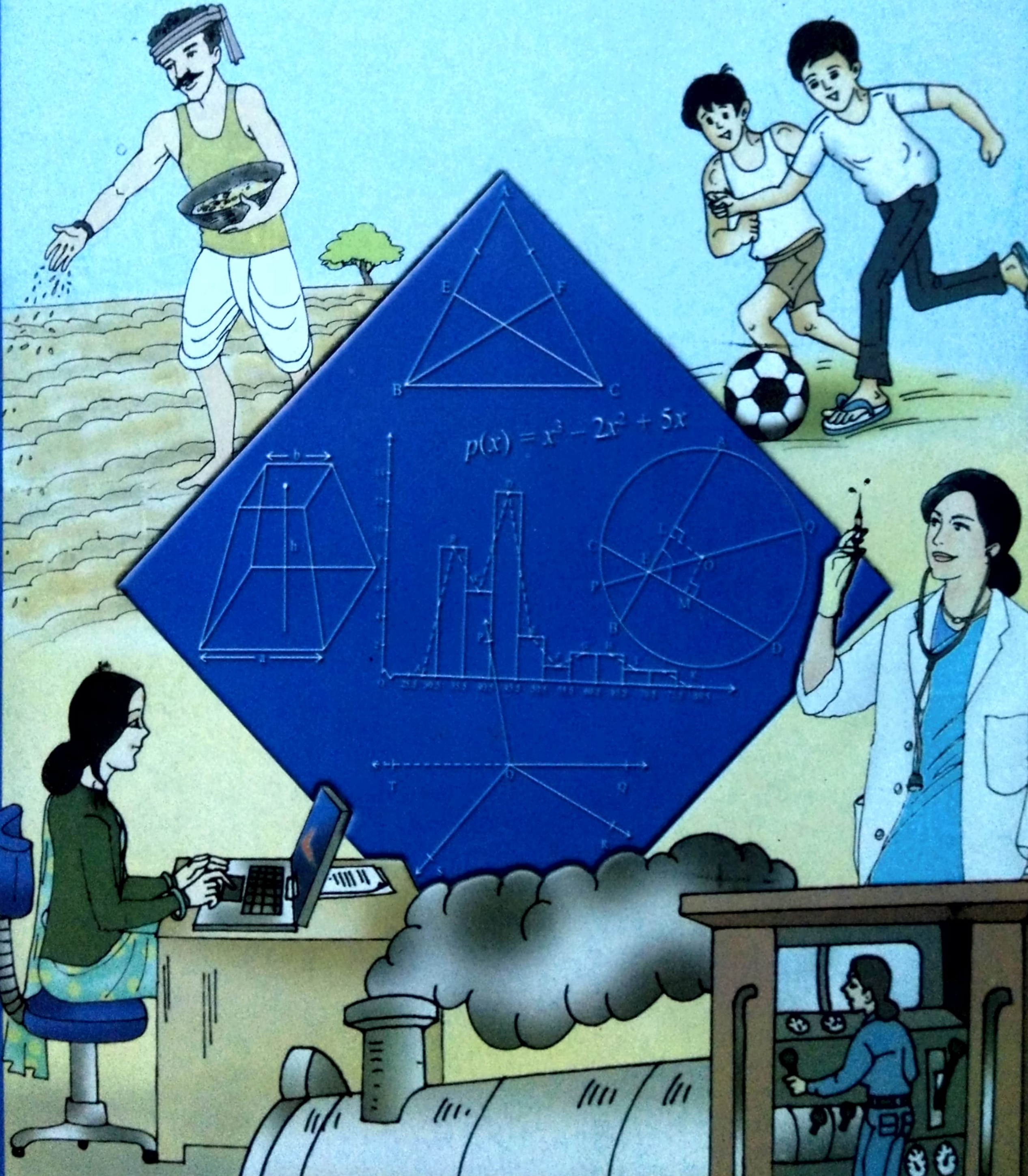
# MATHEMATICS

## Textbook for Class IX



$)^0$? Yes, it is 1! So you have learnt that ... . So, using (iii), we can

(iii) $\frac{23^{10}}{...} = 23^{-17}$ ... (iv) $(7)^{-3} \cdot (9)^{-3} = (63)^{-3}$

(iii) $\frac{7^{\frac{1}{2}}}{7^{\frac{1}{3}}}$ (iv) $13^{\frac{1}{5}} \cdot 17^{\frac{1}{5}}$

How would we go about it? It turns out that we can extend the laws of exponents ... studied earlier, even when the base is a positive real number and the ... further ... extended ... rational ... these laws, and to even ... onents ...

... sense of these laws, we need to first understand what, for example $4^{\frac{3}{2}}$ is. So, ... some work to do!

... fine $\sqrt[n]{a}$ for a real number $a > 0$ as follows:

Let $a > 0$ be a real number and $n$ a positive integer. Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$.

In the language of exponents, we define $\sqrt[n]{a} = a^{\frac{1}{n}}$. So, in particular, $\sqrt[3]{2} = 2^{\frac{1}{3}}$.

There are now two ways to look at $4^{\frac{3}{2}}$.

$$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8$$

$$4^{\frac{3}{2}} = \left(4^3\right)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$$

Let $a > 0$ be a real number and $p$ and $q$ be rational numbers. Then, we have

**Example 20 :** Simplify (i) [illegible] (ii) [illegible]

**Solution :**

(iii) $\frac{7^{[illegible]}}{7^{\frac{1}{3}}}$

## EXERCISE 1.5

1. Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{[illegible]}$ (iii) $12[illegible]$
2. Find : (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{-\frac{1}{3}}$
3. Simplify : (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

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CHAPTER 2

# POLYNOMIALS

## 2.1 Introduction

You have studied algebraic expressions, their addition, subtraction, multiplication and division in earlier classes. You also have studied how to factorise some algebraic expressions. You may recall the algebraic identities :

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$x^2 - y^2 = (x + y)(x - y)$$

and their use in factorisation. In this chapter, we shall start our study with a particular type of algebraic expression, called *polynomial*, and the terminology related to it. We shall also study the *Remainder Theorem* and *Factor Theorem* and their use in the factorisation of polynomials. In addition to the above, we shall study some more algebraic identities and their use in factorisation and in evaluating some given expressions.

## 2.2 Polynomials in One Variable

Let us begin by recalling that a variable is denoted by a symbol that can take any real value. We use the letters $x$, $y$, $z$, etc. to denote variables. Notice that $2x$, $3x$, $-x$, $-\frac{1}{2}x$ are algebraic expressions. All these expressions are of the form (a constant) $\times$ $x$. Now suppose we want to write an expression which is (a constant) $\times$ (a variable) and we do not know what the constant is. In such cases, we write the constant as $a$, $b$, $c$, etc. So the expression will be $ax$, say.

However, there is a difference between a letter denoting a constant and a letter denoting a variable. The values of the constants remain the same throughout a particular situation, that is, the values of the constants do not change in a given problem, but the value of a variable can keep changing.

## EXERCISE 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

   (i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$

   (v) $x^{10} + y^3 + t^{50}$

2. Write the coefficients of $x^2$ in each of the following:

   (i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

4. Write the degree of each of the following polynomials:

   (i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

   (iii) $5t - \sqrt{7}$ (iv) $3$

5. Classify the following as linear, quadratic and cubic polynomials:

   (i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$ (iv) $1 + x$

   (v) $3t$ (vi) $r^2$ (vii) $7x^3$

## 2.3 Zeroes of a Polynomial

Consider the polynomial $p(x) = 5x^3 - 2x^2 + 3x - 2$.

If we replace $x$ by 1 everywhere in $p(x)$, we get

$$\begin{aligned} p(1) &= 5 \times (1)^3 - 2 \times (1)^2 + 3 \times (1) - 2 \\ &= 5 - 2 + 3 - 2 \\ &= 4 \end{aligned}$$

So, we say that the value of $p(x)$ at $x = 1$ is 4.

Similarly, $p(0) = 5(0)^3 - 2(0)^2 + 3(0) - 2$

$= -2$

Can you find $p(-1)$?

**Example 2 :** Find the value of each of the following polynomials at the indicated value of variables:

(i) $p(x) = 5x^2 - 3x + 7$ at $x = 1$.

(ii) $q(y) = 3y^3 - 4y + \sqrt{11}$ at $y = 2$.

(iii) $p(t) = 4t^4 + 5t^3 - t^2 + 6$ at $t = a$.

**Example 5 :**

**Solution :**

## EXERCISE 2.2

Again,

So, $x + 2$ is a factor of $2x + 4$. In fact, you can check this without applying the Factor Theorem, since $2x + 4 = 2(x + 2)$.

**Example 7 :** Find the value of $k$, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$.

**Solution :** As $x - 1$ is a factor of $p(x) = 4x^3 + 3x^2 - 4x + k$, $p(1) = 0$

Now, $\quad p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k$

So. $\quad 4 + 3 - 4 + k = 0$

i.e., $\quad k = -3$

We will now use the Factor Theorem to factorise some polynomials of degree 2 and 3. You are already familiar with the factorisation of a quadratic polynomial like $x^2 + lx + m$. You had factorised it by splitting the middle term $lx$ as $ax + bx$ so that $ab = m$. Then $x^2 + lx + m = (x + a)(x + b)$. We shall now try to factorise quadratic polynomials of the type $ax^2 + bx + c$, where $a \neq 0$ and $a$, $b$, $c$ are constants

Factorisation of the polynomial $ax^2 + bx + c$ **by splitting the middle term** is as follows:

$\frac{3x^2}{x} = 3x$ = first term of quotient

Let its factors be $(px + q)$ and $(rx + s)$. Then

$$ax^2 + bx + c = (px + q)(rx + s) = pr\,x^2 + (ps + qr)\,x + qs$$

Comparing the coefficients of $x^2$, we get $a = pr$.

Similarly, comparing the coefficients of $x$, we get $b = ps + qr$.

And, on comparing the constant terms, we get $c = qs$.

This shows us that $b$ is the sum of two numbers $ps$ and $qr$, whose product is $(ps)(qr) = (pr)(qs) = ac$.

Therefore, to factorise $ax^2 + bx + c$, we have to write $b$ as the sum of two numbers whose product is $ac$. This will be clear from Example 13.

**Example 8 :** Factorise $6x^2 + 17x + 5$ by splitting the middle term, and by using the Factor Theorem.

**Solution 1 :** (By splitting method) : If we can find two numbers $p$ and $q$ such that $p + q = 17$ and $pq = 6 \times 5 = 30$, then we can get the factors.

So, let us look for the pairs of factors of 30. Some are 1 and 30, 2 and 15, 3 and 10, 5 and 6. Of these pairs, 2 and 15 will give us $p + q = 17$.

**Solution 2 :**

17 5 ... the zeroes

**Example 9 :**

**Solution :**

ab. So, $ab = 6$. So, ... look for ... you kn... ), we look at

6 are 1, 2

$-(5 \times \ldots + 6$

Also, $p(3) = 3^2 - (5 \times 3) + 6 = 0$

So, $x - 3$ is also a factor of $x^2 - 5x + 6$.

Now, let us consider factorising cubic polynomials. Here, the splitting method will not be appropriate to start with. We need to find at least one factor first, as you will see in the following example.

**Example 10 :**

**Solution :**

We shall now look for all the factors of $-120$. Some of these are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 60$.

By trial, we find that $p(1) = 0$. So $x - 1$ is a factor of $p(x)$.

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$$= x^2(x - 1) - 22x(x - 1) + 120(x - 1) \quad \text{(Why?)}$$

$$= (x - 1)(x^2 - 22x + 120) \quad [\text{Taking } (x - 1) \text{ common}]$$

We could have also got this by dividing $p(x)$ by $x - 1$.

Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have:

$$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$$

$$= x(x - 12) - 10(x - 12)$$

$$= (x - 12)(x - 10)$$

So, $$x^3 - 23x^2 - 142x - 120 = (x - 1)(x - 10)(x - 12)$$

## EXERCISE 2.3

1. Determine which of the following polynomials has $(x + 1)$ a factor :

   (i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

   (iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

   (i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$